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ICMME 2022

MATHEMATICIANS ASSOCIATION 1995

INTERNATIONAL CONFERENCE ON MATHEMATICS AND MATHEMATICS EDUCATION

22-24 SEPTEMBER 2022

ABSTRACT BOOK

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DENİZLİ BÜYÜKŞEHİR BELEDİYESİ

PAMUKKALE BELEDİYESİ

MAVİNEFES

DENİZLİ KOLEJİ

International Conference on Mathematics and Mathematics Education
(ICMME - 2022)

Pamukkale University, Denizli, Turkey, 22-24 September 2022

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PREFACE

International Conference on Mathematics and Mathematics Education (ICMME-2022) was held on 22-24 September 2022 in Denizli, Turkey. The conference presented new results and future challenges, in series of invited and short talks, poster presentations, workshops, and exhibitions. This year, our conference included a chess tournament, Mathematics and Art Exhibition, and a high school math competition.

MATDER-Association of Mathematicians is an association founded in 1995 by mathematicians in Turkey. Up to now 14 national and 2 international mathematics symposium were organized by MATDER.

These meetings have been one of the main national symposiums. Since the talks in the meetings covers almost all areas of mathematics, mathematics education and engineering mathematics, the conferences have been well attended by mathematicians from academia, Ministry of Education and engineers as well. The last six conferences have been held in Ankara (ICMME-2021), Konya (ICMME-2019), Ordu (ICMME-2018), Şanlıurfa (ICMME-2017), Elazig (ICMME-2016) and Niğde (2015). This year ICMME-2022 has been held at Pamukkele University in Denizli/Turkey on 22-24 September 2022 as an international conference.

The main aim of this conference is to contribute to the development of mathematical sciences, mathematical education, and their applications and to bring together the members of the mathematics community, interdisciplinary researchers, educators, mathematicians, and statisticians from all over the world. The conference will present new results and future challenges, in series of invited and short talks, poster presentations, workshops, and exhibitions. All presented paper's abstracts will be published in the conference proceeding. Moreover, selected and peer review articles will be published in the following journals:

- Turkish Journal of Mathematics & Computer Science (TJMCS)
- Pamukkale University Journal of Education
- Advanced Studies: Euro-Tbilisi Mathematical Journal

This conference is organised by MATDER-Association of Mathematicians and Pamukkale University.

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CONTENTS

INVITED SPEAKERS

Some Interdisciplinary Applications of Fuzzy Logic

Ömer AKIN [1]
1. TOBB ETU, Department of Mathematics, Ankara, Türkiye,
omerakin@etu.edu.tr

ABSTRACT

In this talk we would like to give some important applications of fuzzy logic of which started in 1965 by Lotfi Ali Askerzadeh. Firstly we give some fundamental knowledge for fuzzy logic and its historical developments. An Application of Fuzzified Environment for SAIR Model Using COVID-19 Data in Turkey. Modelling in Health treatment of COVID-19, modelling of phytosociology. Here we extend the classical SAIR model to fuzzified environment. Application of Fuzzy Logic and Fuzzy Similarity in plant sociology. When doing this we get help of strong In recent years, studies on plant communities with fuzzy logic and fuzzy similarity approaches have accelerated. The plant communities (plant associations) detected in studies on different habitats and different vegetation types can be interpreted more realistically by re-evaluating them with fuzzy logic and fuzzy similarity approaches. These studies can be applied on the plant communities identified in the current researches as well as directly in the field studies from the beginning.

Key Words: Fuzzy logic, corona, plant sociology.

Weighted integral inequalities restricted on the cone of monotone functions and applications in the embedding's of Lorentz-type spaces

Amiran Gogatishvili[1],

1 Instititue of Mathematics of the Czech Academy of Sciences
Zitna 25, Prague, Czech Republic
gogatish@math.cas.cz

ABSTRACT

Let ρ be a monotone quasinorm defined on M^+, the set of all non-negativemeasurable functions on $[0,\infty)$. Let T be a monotone quasilinear operator on M^+. We show that the following inequality restricted on the cone of λ-quasiconcaveFunctions

$$\rho(Tf) \leq C_1 (\int_0^\infty f^p v)^{\frac{1}{p}}$$

where $1 \leq p \leq \infty$ and v is a weighted function, is equivalent to slightly differentinequalities considered for all non-negative measurable functions. The case $0 < p < 1$ is also studied for quasinorms and operators with additional properties. These results in turn enable us to establish necessary and sufficient conditions on the weights (u, v, w) for which the three weighted Hardy-type inequality

$$(\int_0^\infty (\int_0^x fu)^q \, w)^{\frac{1}{q}} \leq C_1 \, (\int_0^\infty f^p v)^{\frac{1}{p}}$$

,
holds for all λ-quasiconcave functions and all $0 < p, q \leq \infty$.

We present the applications of these results in the embedding of Lorentz-type spaces and real interpolation theory.

Key Words: Quasilinear operator, integral inequality, Lebesgue space; weight, Hardy operator, quasiconcave functions, monotone functions.

Differences in the Way of Doing Maths from Past to Present

İbrahim Çetin [1]
1 Necmettin Erbakan University, Ahmet Kelesoğlu Education Faculty Maths Education,
ibrahimcetin@erbakan.edu.tr

ABSTRACT

In the increasingly digital world, there are still serious discussions about effective and understandable mathematics education. While mathematicians complain that mathematics is not understood enough, learners complain that it is difficult to learn mathematics. So, is there a way to satisfy both mathematicians and math learners? By moving away from the nature of mathematics, which emerged with the understanding of producing a solution to a problem existing in nature through systematic thinking, it has lost its organic aspect and became increasingly synthetic in today's world by only being associated with numbers, calculations, and formulas. For this reason, this difference in the way we do mathematics needs to be seriously questioned. Technology-supported learning environments and outdoor activities can be a solution for establishing this relationship and for a meaningful mathematics teaching, especially in school mathematics where the real life-mathematics relationship cannot be established sufficiently. In learning environments where technology is used effectively, the way we do mathematics changes and opens the door for more meaningful learning. What are the suggestions we can offer to today's mathematics education for meaningful and permanent learning? What kind of changes can teachers make in their classrooms to overcome this problem, who are in the conflict of teaching mathematics by solving a lot of questions and raising a curriculum? In this presentation, the differences in the way we do mathematics from past to present and what we need to do for an effective mathematics teaching today will be explained with different examples. Discussions will be made on the contribution of technology-supported and effective mathematics teaching, which is associated with real life, to mathematics education.

Key Words: Doing maths, technology, outdoor maths education

On addition formulas related to elliptic integrals

Malkhaz Bakuradze[1]
1 Tbilisi State University, faculty of exact and natural sciences, 0167, Tbilisi, Georgia
malkhaz.bakuradze@tsu.ge

ABSTRACT

Addition theorems offer a means of determining the value of the function for the sum of two quantities as arguments, when the values of the function for each argument is known. The simplest examples are the well-known addition formulas for the functions e^u and $\sin u$.

The elliptic sine $f(u) = sn(u)$, satisfies Euler's addition theorem

$$f(u+v) = \frac{f(u)R(f(v)) + f(v)R(f(u))}{1-k^2 f(u)^2 f(v)^2}, \quad \text{where} \quad R(u) = \sqrt{(1-u^2)(1-k^2u^2)}$$

and k is some parameter called modulus.
We review some other interesting examples, such as Baker-Akhiezer functions and the exponent series of the Ochanine elliptic genus.

As a main result we provide certain addition formulas for the elliptic integrals corresponding to the general elliptic genus with logarithm having differential $\frac{dt}{dR(t)}$, where $R(t)$ is a monic polynomial of degree 4. Our result specializes in Euler's addition theorems for elliptic integrals of the first and second kind and in the examples above.

The proofs are given in terms of the formal group laws.

Key Words: Formal group law, elliptic integral.

Teaching proof with an interactive theorem prover

Gila Hanna[1] and Xiaoheng (Kitty) Yan[2]
1 University of Toronto, Department of Curriculum, Teaching and Learning
2 University of Toronto, Department of Curriculum, Teaching and Learning
gila.hanna@utoronto.ca
xkyan@sfu.ca

ABSTRACT

Let A and B be two abelian groups. The group A is called B-small if the covariant functor Hom(A,-) commutes with all direct sums of the form B(k) and A is a self-small group provided it is A-small. The main aim of the talk is to characterize self-small products applying developed closure properties of the classes of relatively small groups. In particular, we show that a product of a system of abelian groups is self small if and only if it relatively small over a direct sum of the system.

As a consequence of the theory of relatively small groups and the well-known fact that powers Zk of the group Z of all integers is slender for any nonmeasurable cardinal k, we characterize self-small products of finitely generated abelian groups. Namely, the product M of finitely generated groups is self-small if and only if either M is isomorphic to power Zk for some cardinal k, or M is isomorphic to a direct sum of a finitely generated free group F and finite abelian p-groups for each prime number p.

Finally, we also discuss possible application of the developed tools for description of self-compact objects in context of general additive and abelian categories.

Weighted Morrey spaces

Yoshihiro Sawano[1]
1 1-3-1-625, Kasuga, Bunkyo-ku, 112-8551, Tokyo Japan

ABSTRACT

The goal of this talk is to consider the boundedness property of sublinear operators. In 2009, weighted Morrey spaces are considered independently by Natasha Samko and by Yasuo Komori-Furuya and Satoru Shirai. One of the standard assumptions on weights is that they belong to the Muckenhoupt class. However, it is too strong in some cases. In this talk, I address the problem by the use of the power weights. In the case of local Morrey spaces, we can completely characterize the boundedness property for weighted Morrey spaces of Samko type.

Key Words: Morrey spaces, Muckenhoupt weights, power weights.

ABSTRACTS OF ORAL PRESENTATIONS

ALGEBRA AND NUMBER THEORY

New Coding/Decoding Algorithm on k-Order Fibonacci Numbers

CANAN CELEP YUCEL [1], MUSTAFA ASCI [2], AND SEREL MADAK [3]

1 Pamukkale University Science and Arts Faculty Department of Mathematics Kinikli 20160 Denizli TURKEY

ccyucel@pau.edu.tr

2 Pamukkale University Science and Arts Faculty Department of Mathematics Kinikli 20160 Denizli TURKEY

mustafa.asci@yahoo.com

3 Pamukkale University Science and Arts Faculty Department of Mathematics Kinikli 20160 Denizli TURKEY

serenmadak-96@outlook.com

ABSTRACT

In this paper, we define a new encryption algorithm by generalizing the 2 x 2 encryption algorithm and dividing it into k x k type block matrices. This coding algorithm bound to the Q_k matrices. Therefore, this algorithm is different from the classical algebraic algorithm. Since matrix operations are at the forefront in this algorithm, it is easily and quickly performed by todayís modern computers.

Key words: Fibonacci numbers, k-order Fibonacci numbers, Coding/decoding method, k-order Fibonacci numbers coding/decoding Algorithm.

REFERENCES

[1] Koshy T., "Fibonacci and Lucas Numbers with Applications", A Wiley-Interscience Publi- cation, (2001).

[2] Vajda S., "Fibonacci and Lucas Numbers and the Golden Section Theory and Applications", Ellis Harwood Limitted, (1989).

[3] Gould H. W., "A history of the Fibonacci Q-matrix and a Higher-Dimensional Problem", Fibonacci Quart. 19 (1981), no. 3, 250-257.

[4] Hoggat V. E., "Fibonacci and Lucas Numbers", Houghton-Mi in, Palo Alto (1969).

[5] Stakhov A. P., "A generalization of the Fibonacci Q-matrix", Rep. Natl. Acad. Sci. Ukraine 9 (1999), 46-49.

[6] Stakhov A. P., Massinggue V., Sluchenkov A., "Introduction into Fibonacci Coding and Cryptography", Osnova, Kharkov (1999).

[7] Er, M. C. "Sums of Fibonacci numbers by matrix methods." Fibonacci Quart 22.3 (1984): 204-207.

[8] Stakhov A. P., "Fibonacci matrices, a generalization of the Cassini formula and a new coding theory", Chaos, Solitions and Fractals 30 (2006), no. 1, 56-66.

[9] Basu M., Prasad B., "The generalized relations among the code elements for Fibonacci Coding Theory", Chaos, Solitons and Fractals 41(5) (2009), 2517,2525.

[10] Basu M., Das M., "Tribonacci matrices and a new coding theory", Discrete Math. Algorithms Appl., 6(1), (2014) article ID: 1450008.

[11] Basu M., Das M., "Coding theory on Fibonacci n-step numbers", Discrete Math. Algorithms Appl., 6(2), (2014) article ID: 145008.

[12] Asci M., Aydinyuz S., "k-order Gaussian Fibonacci Polynomials and Applications to the coding/decoding theory", Journal of Discrete Mathematical Sciences & Cryptography, In press, DOI : 10.1080/09720529.2020.1816917.

[13] Ucar S., Tas N., Ozgur N. Y., "A new Application to Coding Theory via Fibonacci and Lucas Numbers", Mathematical Sciences and Applications E-Notes, 7(1), 62-70, (2019).

Moore-Penrose Inverse on Generalized Fibonacci Matrix

Suleyman AYDINYUZ [1], Mustafa ASCI[2]
1 Pamukkale University, Science Faculty, Department of Mathematics
aydinyuzsuleyman@gmail.com
2 Pamukkale University, Science and Arts Faculty, Department of Mathematics
mustafa.asci@yahoo.com

ABSTRACT

In this paper, we define the Moore-Penrose inverse on generalized Fibonacci matrix. This matrix $U \in M_{m,n}(C)$, whose elements are Fibonacci numbers, is a rectangular matrix. We consider $U_n = pU_{n-1} - U_{n-2}$ with the initial conditions $U_0 = 0, U_1 = 1$ generalized Fibonacci numbers and define the generalized Moore-Penrose inverse. We get the special numbers according to the choices depending on p. When we choose $p=1$ and $p=2$, we get Fibonacci and Pell numbers, respectively. Therefore, depending on these particular choices, Moore-Penrose inverses of rectangular Fibonacci and Pell matrices can be found.

Key Words: Fibonacci Matrix, The Moore-Penrose Inverse, Pseudoinverse, Generalized Fibonacci matrix.

REFERENCES

[1] Vajda, S., Fibonacci & Lucas numbers, and the golden section, Theory and applications, John Wiley & Sons, New York, 24-61, 1989.

[2] Koshy, T., Fibonacci and Lucas Numbers with Applications, Wiley-Interscience, 2001.

[3] Moore, E. H., On the reciprocal of the general algebraic matrix, Bulletin of the American Mathematical Society, (1920), 26 (9): 394--95. doi:10.1090/S0002-9904-1920-03322-7.

[4] Penrose, R., A generalized inverse for matrices, Proceedings of the Cambridge Philosophical Society, (1955), 51 (3): 406--13, doi:10.1017/S0305004100030401.

[5] Boman, E. C., The Moore--Penrose pseudoinverse of an arbitrary, square k-circulant matrix. Linear Multilinear Algebra, (2002), 50(2): 175—179.

[6] Fill, J. A. and Fishkind, D. E., The Moore--Penrose generalized inverse for sums of matrices. SIAM J. Matrix Anal. Appl., (2000), 21(2): 629—635.

[7] Schmidt, K. and Trenkler, G., The Moore--Penrose inverse of a semi-magic square is semi-magic. Int. J. Math. Educ. Sci. Technol., (2001), 32(4): 624—629.

[8] Israel, B. A., The Moore of the Moore-Penrose Inverse. The Electronic Journal of Linear Algebra, (2002), 9, 150-157.

Idun-semicommutative Rings

Miraç Çetin [1], M. Tamer Koşan [2] and Jan Zemlicka[3]
1 Department of Mathematics Educations, Baskent University University, Ankara, Turkey, mcetin@baskent.edu.tr
2 Department of Mathematics, Faculty of Sciences, Gazi University, Ankara, Turkey tkosan@gmail.com
3 Department of Algebra, Charles University in Prague, Faculty of Mathematics and Physics Sokolovská, 83, 186 75 Praha 8, Czech Republic
zemlicka@karlin.mff.cuni.cz

ABSTRACT

According to Calugareanu [1], R is said to be a *unit-semiprime* ring if, for any $a \in R$, $aU(R)a = 0$ implies $a = 0$, where $U(R)$ is the group of units of R.

In this presentation, we talk about the following equivalent conditions, which is called *idun-semicommutative* rings by Çetin-Koşan-Zemlicka [2]; if $xy = 0$ implies $x\big(Id(R) + U(R)\big)y = 0$ for all $x, y \in R$ iff $\big(Id(R) + U(R)\big)ann_r(x)(x) \subseteq ann_r(x)$ for all $x \in R$ (in view of [3]).

Key Words: Semiperfect ring, (unit-)semiprime ring, (unit-)semicommutative ring

REFERENCES

[1] G. Calugareanu, A new class of semiprime rings, Houston J. Math. 44 (2018), 21-30.

[2] M. Çetin, M. T. Kosan, J. Zemlicka, On rings determined by their idempotents and units, Communications in Algebra, in press.

[3] M. T. Kosan, T.C. Quynh, On rings with associated elements, Communications in Algebra, 45 (7) (2017), 2747-2756.

Catalan Numbers in terms of (α,k)-gamma function and (α,k)-beta function

Abdullah Akkurt[1], and Hüseyin Yıldırım[1]

1 Department of Mathematics, Faculty of Science, Kahramanmaraş Sütçü İmam University, 46100, Kahramanmaraş, Turkey,
abdullahmat@gmail.com

2 Department of Mathematics, Faculty of Science, Kahramanmaraş Sütçü İmam University, 46100, Kahramanmaraş, Turkey,
hyildir@ksu.edu.tr

ABSTRACT

The first few Catalan numbers C_n for $0 \leq n \leq 14$ are;

$1,\ 1,\ 2,\ 5,\ 14,\ 42,\ 132,\ 429,\ 1430,\ 4862,\ 16796,\ 58786,\ 208012,\ 742900,\ 2674440.$

The Catalan numbers are defined by means of the following generating functions;

$$\frac{1-\sqrt{1-4x}}{2x} = \sum_{n=0}^{\infty} C_n x^n$$
$$= 1 + x + 2x^2 + 5x^3 + 14x^4 + 42x^5 \ldots$$

One of explicit formulas of C_n for $n \geq 0$ reads that

$$C_n = \frac{4^n \Gamma\left(n+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(n+2)} = \frac{1}{n+1}\binom{2n}{n}.$$

For more information on the Catalan numbers C_n , please see ([4-6, 8, 9]).

In this work, we will give some generalized results for the C_n numbers known as Catalan numbers in the literature. We will give some new formulas for Catalan numbers, their integral representation, and a parametric integral notation such as the (α,k) –gamma and (α,k) –beta functions [1-3, 7]. We will show that the obtained results with their special selection give the current results in the literature.

Key Words: Catalan number, (α,k) –gamma function, (α,k) –beta function.

REFERENCES

[1] Andrews, G. E., Askey, R., Roy, R., Roy, R., & Askey, R. (1999). Special functions (Vol. 71, pp. xvi+-664). Cambridge: Cambridge university press.

[2] Comtet, L. (2012). Advanced Combinatorics: The art of finite and infinite expansions. Springer Science and Business Media.

[3] Sarıkaya M. Z., Akkurt A., Budak H., Türkay M. E., Yildirim H. (2020). On Some Special Functions for Conformable Fractional Integrals. Konuralp J. Math. 8(2): 376-383.

[4] Koshy, T. (2008). Catalan numbers with applications. Oxford University Press.

[5] Qi, F. (2017). Parametric integrals, the Catalan numbers, and the beta function. Elemente Der Mathematik, 72(3), 103-110.

[6] Qi, F., Akkurt, A., and Yildirim, H. (2018). Catalan numbers, k-gamma and k-beta functions, and parametric integrals. J. Comput. Anal. Appl, 25(6), 1036-1042.

[7] Lebedev, N. N., Silverman, R. A., and Livhtenberg, D. B. (1965). Special functions and their applications. Physics Today, 18(12), 70.

[8] Shi, X. T., Liu, F. F., & Qi, F. (2015). An integral representation of the Catalan numbers. Glob. J. Math. Anal, 3(3), 130-133.

[9] Dana-Picard, T. (2005). Parametric integrals and Catalan numbers. International Journal of Mathematical Education in Science and Technology, 36(4), 410-414.

On Bivariate Gaussian Balancing Polynomials

Mustafa YILMAZ[1] and Mustafa AŞCI

1 Pamukkale University, Science Faculty, Department of Mathematics
2 Pamukkale University, Science Faculty, Department of Mathematics
mustafa.asci@yahoo.com

ABSTRACT

In this paper we define and study the bivariate Gaussian balancing and Gaussian cobalancing polynomials. We identify the Binet formula, generating function, summation formulas, matrix and determinant representations of these polynomials. By this study we generalize the polynomials defined before this study.

Key Words: Fibonacci numbers, Balancing numbers, Gauss balancing numbers

REFERENCES

[1] Asci M., Gurel E., "Bivariate Gaussian Fibonacci and Lucas polynomials", Ars Comb., 109(2013): 461-472.

[2] Asci M., Gurel E.,"Gaussian Jacobsthal and Gaussian Jacobsthal Lucas numbers", Ars Comb., 111(2013): 53-63.

[3] Asci M., Gurel E., "Gaussian Fibonacci and Gaussian Lucas p-numbers", Ars Comb., 132(2017): 389-402.

[4] M. Asci, M. Yakar "On Bivariate Balancing Polynomials*" JP J. Algebra Number Theory Appl.* 46, (2020), No. 1, 97-108

[5] Behera, A., Panda, G. K., "On the square roots of triangular numbers", Fibonacci Quart., 37(1999), 98-105.

[6] Horadam, A. F., "A Generalized Fibonacci sequence." American Math. Monthly, 68(1961): 455-459.

[7] Jordan, J. H., "Gaussian Fibonacci and Lucas numbers". Fibonacci Quart., 3(1965) 315-318.

[8] Panda, G. K., Ray, P. K., "Cobalancing numbers and cobalancers", Int. J. Math. Sci., 8(2005): 1189-1200.

[9] Panda, G. K., "Some fascinating properties of balancing numbers", Congr. Numer., 194(2009): 185-189.

Permuting n-(f,g)-Derivations On Lattices

Şahin CERAN [1] and Utku Pehlivan[2]
1 Pamukkale University Science Faculty
sceran@pau.edu.tr
2 Pamukkale University Science Faculty
utkuphlvn@gmail.com

ABSTRACT

Ceven and Ozturk gave some results of symmetric bi-(σ,τ)-derivation of near rings in [1]. Also in [2] Ceven studied this derivation structure for lattices. In [3] Ozturk and Yazarlı introduced permuting tri-(f,g)-derivation notion and gave some properties in lattices. Also in [5] Alshehri defined generalized derivations of lattices and discussed related properties.Then Chaudry and Ulah studied at generalized (α,β) derivations on lattices in [6]. Ascı and Ceran in [4] studied symmetric bi-(σ,τ)-derivation of prime near rings.They also proved some results for permuting tri-(f,g)-derivations on lattices in [7] and generalized (f,g) derivation on lattices in [8]. In this paper as a generalization of permuting tri-(f, g)-derivation on a lattice we introduced the notion of permuting n-(f, g)-derivation of a lattice. We defined the isotone permuting n-(f,g)-derivation and got some interesting results about isotoneness. We characterized the distributive and isotone lattices by permuting n-(f,g)-derivation. About these derivations, detailed informations are also given. Finally we gave some results about properties of f,g functions on lattices. We discussed this new structure relationship with generalized (f,g) derivations on lattices.

Key Words: n-(f,g)-derivation, Permuting mapping, Distributive lattice, İsotone lattice

REFERENCES

[1] Ceven, Y., Ozturk, M. A., Some properties of symmetric bi-(σ,τ) -derivations in near-rings. Commun. Korean Math. Soc. 22 (2007), no. 4, 487-491.

[2] Ceven, Y., Symmetric bi derivations of Lattices. Quaestiones Mathematicae, 32(2009), 1-5.

[3] Ozturk, M.A., Yazarlı, H., Kim, K.H., Permuting tri-derivations in lattices. Quaest. Math. 32 (2009), no. 3, 415-425.

[4] Ascı, M., Ceran, S., Symmetric bi -(σ,τ)- derivations of Prime Near Rings, Algebras,Groups, Geometry, 24 (2007), no. 3, 291-302.

[5] Alshehri, N.O., Generalized derivations of Lattices Int.J. Contemp. Math. Sci.,Vol 5, 2010, no. 13, 629-640.

[6] Chaudry, M.A.,Ulah, Z., On generalized (α,β) derivations on lattices, Quaestiones Mathematicae,34(2011),417-424.

[7] Ascı, M., Kecilioglu, O., Ceran, S., Permuting tri-(f,g)-derivations on lattices, Annals of Fuzzy Mathematics and Informatics 1 (2011), No. 2, 189-196.

[8] Ascı, M., Ceran, S., Generalized (f, g)-derivations of lattices, Math. Sci. and App. E- Notes Vol. 1(2013),No. 2, pp 56-62.

Permuting n-Derivations From Semilattices to Lattice

Şahin CERAN [1] and Utku PEHLİVAN[2]
1 Pamukkale University, Science Faculty, Department of Mathematics
sceran@pau.edu.tr
2 Pamukkale University, Science Faculty, Department of Mathematics
utkuphlvn@gmail.com

ABSTRACT

Szasz introduced the notion of lattice derivation and gave interesting results in [1]. Also in [2] the author studied this lattice derivation. In [3] Xin and et al. improved derivation for a lattice and discussed some related properties. They gave some equivalent conditions under which a derivation is isotone for lattices with a greatest element, modular lattices and distributive lattices. In [5] Ozturk introduced the notion of permuting tri-derivations in rings and proved some results. Also in [4] Ozturk and et al. introduced the permuting tri-derivations in lattices. In this paper the definitions of derivations which are defined in lattices are given. About these derivations, detailed informations are also given. In the first chapter the definitions are some basic theorems are which will be used in the other chapters. In the second chapter the definitions, some properties and theorems of the derivations on lattices are given without proof. In the third chapter the definitions of permuting n- derivations are given.In this paper as a generalization of permuting tri-derivation on a lattice. We introduced the notion of permuting n- derivation from semilattices to lattice. We defined the isotone permuting n-derivation from semilattices to lattice and got some interesting results about isotoneness. We characterized the distributive and isotone lattices by permuting n-derivation from semilattices to lattice.

Key Words: Lattices, Derivation, semilattices

REFERENCES

[1] Szasz, G. Derivations of lattices. Acta Sci. Math. (Szeged) 37 (1975), 149-154.

[2] Ferrari, Luca On derivations of lattices. Pure Math. Appl. 12 (2001), no. 4, 365-382.

[3] Xin, X. L., Li, T. Y., and Lu, J. H. On derivations of lattices, Inform. Sci. 178 (2008), no. 2, 307-316

[4] Ozturk, Mehmet Ali; Yazarlı, Hasret; Kim, Kyung Ho Permuting tri-derivations in lattices. Quaest. Math. 32 (2009), no. 3, 415-425

[5] Ozturk, Mehmet Ali., Permuting Tri-derivations is Prime and Semi-prime Rings, East Asian Math. J. 15(2) (1999), 177-190.

Bivariate Lucas Balancing Polynomials

Mustafa AŞCI[1], Merve YAKAR[2]
1 Pamukkale University, Science Faculty, Department of Mathematics
mustafa.asci@yahoo.com
2 Pamukkale University, Science Faculty, Department of Mathematics
myakar13@posta.pau.edu.tr

ABSTRACT

In this study we defined the bivariate This file provides a template for writing abstract for the conference. The abstract file shall be written in compliance wlucas balancing polynomials with boundary conditions. We identify and prove Binet formulas, generating functions for bivariate lucas balancing polynomials. And we study some identities between bivariate balancing polynomials and bivariate lucas balancing polynomials

Key Words: bivariate balancing balancing polynomials, bivariate lucas balancing polynomials

REFERENCES

[1] Aşcı,M. And Yakar, M. ‘’ Bivariate Balancing Polynomials’’ JP Journoul Algebra Number Theory and Applications, (2020) 46(1), 97-108

[2] A. Behera and G. K. Panda, "On the Square Roots of Triangular Numbers" Fibonacci Quart., 37 (1999), 98-105.

[3] Panda, G.K., Some Fascinating Properties of Balancing Numbers, Proceedings of the Eleventh International Conference on Fibonacci Numbers and Their Applications, Cong. Numer, 194 (2009) 185-189

[4] Ray, P. K., Panda, G. K., Balancing and Cobalancing Numbers. PhD Thesis, Department of Mathematics, National Institute of Technology, Rourkela, India, (2009)

[5] Panda, G.K., Sequence Balancing and Cobalancing Numbers, The Fibonacci Quaterly, 45 (2007) 265-271

[6] Panda, G.K., Ray, P.K., Cobalancing Numbers and Cobalancer, Int. J. Math. Sci., No. 8 (2005) 1189-1200

[7] Bérczes, A., Liptai, K., Pink, I., On Generalized Balancing Sequences, The Fibonacci Ouaterly, 48, (2) (2010) 121-128

[8] Olajos, P., Properties of Balancing, Cobalancing and Generalized Balancing Numbers, Annales Mathematicae et Informaticae 37 (2010) 125-138

[9] Panda, G.K., Ray, P.K., Some Links of Balancing and Cobalancing Numbers with Pell and Associated Pell Numbers, Bulletin of the Institute of Mathematics, Academia Sinica (New Series), 2011, 6(1): 41-72.

[10] P.K.Ray and G.K.Panda,"Balancing and cobalancing numbers"

Characterization of the Wreath Product $D_{2n} \wr \mathbb{Z}_2$ in Terms of p-Cockcroft Property

Siyat Nur Noyan [1], Ahmet Sinan Çevik [2]

1 Selçuk University, Konya, Turkey
siyatnur5@hotmail.com

2 Selçuk University, Konya, Turkey
sinan.cevik@yahoo.com

ABSTRACT

It is known that when we manage to plot spherical pictures over groups defined by presentations, one may take an advantage to investigate its *p*-Cockcroft property by counting the positive and negative discs in spherical pictures. This gives a distinguish characterize over these groups specially in the meaning of their minimalities. The other way around these studies obtained by some other authors with the homological terminology efficiency of groups. By taking into account a special wreath product $D_{2n} \wr \mathbb{Z}_2$, we will talk about the minimality and so *p*-Cockcroft property by drawing the spherical pictures over them as a new result.

Key Words: Wreath product, p-Cockcroft property, Presentation.

REFERENCES

[1] F. Ates, A.S. Çevik, Separability and Efficiency under Standard Wreath Products in terms of Cayley Graphs, Rocky Mountain Journal of Mathematics, 38(3), (2008), 779-800.

[2] A.S. Çevik, The Efficiency of Standard Wreath Product, Proceeding of the Edinburgh Mathematical Society, 43(2), (2000), 415-423.

[3] A.S. Çevik, The p-Cockcroft property of Semi-Direct Products of Monoids, International Journal of Algebra and Computation, 13(1), (2003),1-16.

A Note on the Balancing-like and Lucas-balancing-like Sequences

Gül Karadeniz Gözeri[1]

1. Istanbul University, Faculty of Science, Department of Mathematics, Istanbul, Turkey
gulkaradeniz@istanbul.edu.tr

ABSTRACT

In this study, we deal with a class of second order linear recurrence sequences defined by $x_{n+1} = Ax_n - x_{n-1}$, $x_0 = 0$, $x_1 = 1$ and $A > 2$ is any natural number. These sequences are called by balancing-like sequences since the sequence of balancing numbers is a particular case of the balancing-like sequences with $A = 6$.

If x is a balancing-like number then $Dx^2 + 1$ is a perfect square where $D = \frac{A^2 - 4}{4}$ and positive square root of $Dx^2 + 1$ is called a Lucas-balancing-like number.

We present firstly, Binet's formulas and some important properties of the balancing-like numbers and Lucas-balancing-like numbers. Then we examine the relations between balancing-like numbers and Lucas-balancing-like numbers. Further, we observe that Lucas-balancing-like numbers are associated with balancing-like numbers in the same way as Lucas balancing numbers associated with balancing numbers.

Key Words: Balancing sequence, balancing-like sequences, Binet formula.

REFERENCES

[1] A. Behera and G. K. Panda, On the square roots of triangular numbers, Fib. Quart. 37 (1999), 98–105.

[2] G.K. Panda and S.S Rout, A Class of recurrent Sequences Exhibiting Some Exiting Properties of Balancing Numbers, Int. J. Math. Comp. Sci. Eng., 6 (2012), 4–6.

[3] G. K. Panda and S. S. Pradhan, Associate sequences of balancing-like sequence, Math. Rep., 22 (72) (2020), 277–291.

Small, compact or connected objects?

Josef Dvořák[1] and Jan Žemlička [2]
1 Department of mathematics, CTU in Prague, FEE,
pepa.dvorak@post.cz
2 Department of Algebra MFF UK, Praha,
zemlicka@karlin.mff.cuni.cz

ABSTRACT

An object over which the corresponding covariant hom-functor commutes with arbitrary coproducts is usually called *small* in categories of modules over a ring [1], *compact* in abelian categories [5], and *connected* in general categories [3]. While compact objects replace the module-theoretical concept of finitely generated objects [1], the connected objects in (non-abelian) category of acts over a monoid generalize the concept of cyclic objects [3].

The aim of the talk is to compare other categorial and structural properties of such objects in several classes of both abelian and (concrete) non-abelian categories. We present in which categories closure properties of the class of small (compact, connected) objects persist to hold. In particular, we describe categorical and set-theoretical conditions under which all products of compact objects remain compact. As projectivity appears to be a useful tool for study of compactness (connectedness), we try to describe structure of projective objects in studied categories. Furthermore, it is proved that projective modules are necessarily small as well as projective acts are connected. Since autocompact objects, whose covariant home-functor commutes only with coproducts of copies of the same object, play important role in abelian case, we describe the corresponding notion of an autoconnected object in particular non-abelian categories.

Key Words: small module, compact object, connected object.

REFERENCES

[1] Arnold, D.M., Murley, C.E., Abelian groups, A, such that Hom(A, -) preserves direct sums of copies of A. Pacific Journal of Mathematics, Vol. 56 (1975), No.1, 7--20.

[2] Dvořák, J. and Žemlička, J., Autocompact objects of Ab5 categories, Theory and Applications of Categories, Vol. 37 (2021), 979-995.

[3] Dvořák, J. and Žemlička, J., Connected objects in categories of S-acts, to appear in Semigroup Forum.

[4] Gomez Pardo, J. L., Militaru, G., Nastasescu, C., When is HOM(M,-) equal to Hom(M,-) in the category R-gr?, Comm. Algebra, 22 (1994), 3171--3181.

[5] Kálnai, P. and Žemlička, J., Compactness in abelian categories, J. Algebra, 534 (2019), 273-288 Boston, MA, 2008.

On Alternative Solutions of Some Differential Equations in Bicomplex Space

Serpil Halıcı [1] and Sule Curuk [2]
1 Pamukkale University, Department of Mathematics, Denizli, 20100, Turkey
email: shalici@pau.edu.tr
2 Pamukkale University, Department of Mathematics, Denizli, 20100, Turkey
email: sule9220@gmail.com

ABSTRACT

Integral transformations are used in the solutions of many boundary-value, initial-value problems, differential equations and integral equations encountered in applied mathematics and engineering. Based on this, new integral transformations are defined and existing integral transformations are expanded. In this study, we gave the bicomplex Fourier transform and bicomplex Hartley transform, which we studied before, based on the studies on integral transforms in bicomplex space. With the help of these transformations, we obtained alternative solutions of the well-known Bessel differential equation. We gave the definition and some basic properties of Hilbert transform. We also examined the relationship between the bicomplex Fourier transform and the bicomplex Hartley transform of the Hilbert transform. Then, using the definition of the Hilbert transform, we found another solution of the Bessel differential equation with the help of the bicomplex Fourier transform and the bicomplex Hartley transform of the Hilbert transform.

Key Words: Integral Transformations; Bicomplex Functions; Bessel Differential Equation.

REFERENCES

[1] A. Banerjee, S. K. Datta and M. A. Hoque, “Fourier transform for functions of bicomplex variables”, arXiv preprint arXiv:1404.4236, 2014.

[2] H. O. Be´ca, “An orthogonal set based on bessel functions of the first kind/skup ortogonalnih funkcija zasnovan na besselovim funkcijama prve vrste”, Publikacije Elektrotehniˇckog fakulteta., Serija Matematika i fizika no. 678/715, (1980), pp. 85–90.

[3] I. Celik, “Numerical solution of differential equations by using chebyshev wavelet collocation method”, Cankaya university journal of science and engineering vol. 10 no. 2,(2013), pp. 169–184.

[4] Johansson, M., “The hilbert transform”, Mathematics Master’s Thesis. Växjö University, Suecia. Disponible en internet: http://w3. msi. vxu. se/exarb/mj_ex. pdf, consultado el, 19, 1999.

[5] M. P. Goswami, “Bicomplex integral transforms and applications”, Ph.D. thesis MNIT Jaipur 2017.

[6] P. Usta Puhl, “Hartley transformations and their applications”, Master Thesis Graduate School of Natural and Applied Sciences, Istanbul, 2016.

[7] R. V. Hartley, “A more symmetrical fourier analysis applied to transmission problems”, Proceedings of the IRE, vol. 30 no. 3, (1942), pp. 144–150.

[8] S. Halıcı and S. Curuk, “On integral transforms of some special functions”, in: Conference Proceedings of Science and Technology, vol. 3 no. 2, (2020), pp. 215–221

p,q-Harmonic Numbers

Serpil Halıcı [1], Zehra Betül Gür [2]
1 Department of Mathematics, Pamukkale University, Denizli, Turkey,
shalici@pau.edu.tr
2 Department of Mathematics, Pamukkale University, Denizli, Turkey,
gurbetul@gmail.com

ABSTRACT

In this study, we examined a new generalization of well-known number sequence which is called harmonic numbers. We defined p,q-harmonic numbers which is also a generalization of q-harmonic numbers and deduced some properties and identities related to this number sequence by using some combinatorial operations.

Firstly we gave definitions of q-analogue and p,q-analogue of and integer and some properties. Also we gave definition of q-harmonic numbers and some special studies concerning this number sequences. After these informations, we defined four types of p,q-harmonic numbers which includes alternating p,q-harmonic numbers. In this study, we examined the finite summations of this number sequences and obtained important results. We gave the finite sum of p,q-analogues of integers from 1 to n. Moreover, we gave two main results includes summations constructed by using some coefficients depending on p and q and p,q-harmonic numbers. We gave some important Lemmas helping to prove our main results.

Key Words: harmonic numbers, q-harmonic numbers, p,q-analogue.

REFERENCES

[1] C. Kızılateş and N. Tuglu, Some Combinatorial Identities of q-Harmonic and q-Hyperharmonic Numbers, Commun. Math. Appl. 6(2)(2015), 33-40.

[2] L. Comtet, Advanced Combinatorics, Reidel, Dordrecht, The Netherlands, 1974.

[3] N. Ömür, Z.B. Gür and S. Koparal, Congruences with q-generalized Catalan numbers and q-harmonic numbers, Hacet. J. Math. Stat. 51(3) (2022), 712-724.

[4] R. Corcino, On p,q-Binomial Coefficients, Integers, 8 (2008), A29.

[5] V. Kac, P. Cheung, Quantum Calculus, Springer, New York, 2002.

ANALYSIS

Some estimates of non-increasing rearrangement of generalized fractional maximal function and cones generated by them

Amiran Gogatishvili [1], Nurzhan Bokayev [2] and Azhar Abek[3]
1 Institute of Mathematics of the Czech Academy of Sciences, Prague, Czech Republic, email: gogatish@math.cas.cz
2 L.N. Gumilyov Eurasian National University, Nur-Sultan, Kazakhstan, email: bokayev2011@yandex.ru
3 L.N. Gumilyov Eurasian National University, Nur-Sultan, Kazakhstan, email: azhar.abekova@gmail.com

ABSTRACT

In the work defines a generalized fractional maximal function (GFMF) $M_\Phi f$ and the spaces generated by it. We give a sharp pointwise estimates of the non-increasing rearrangement of the GFMF. Three types of cones are determined for non-increasing rearrangement of the GFMF and their equivalence is proved.

Key Words: Generalized fractional maximal function, non-increasing rearrangements, cones.

REFERENCES

[1] C.Bennett and R.Sharpley, Interpolation of operators, Pure and applied mathematics 129, Boston, MA: Acad. Press Inc., 1988.

[2] Goldman M.L., On the cones of rearrangements for generalized Bessel and Riesz potentials, Complex Variables and Elliptic Equations 55:8-10 (2010), 817-832.

[3] Goldman M.L., Optimal embeddings of generalized Bessel and Riesz potentials, Trudy Matematicheskogo Instituta imeni V.A. Steklova 269 (2010), 91-111.

[4] Bokayev N. A., Goldman M. L., Karshygina G. Zh., Cones of functions with monotonicity conditions for generalized Bessel and Riesz potentials, Math.Notes 104 (2018), 356–373.

On (p,q)- Analogue of Mittag-Leffler Operators

Nilay ŞAHİN BAYRAM [1], **Reyhan CANATAN İLBEY**[2]
1 Department of Electrical and Electronics Engineering, Faculty of Engineering, Başkent University,Ankara, Turkey, nsbayram@baskent.edu.tr
2 Kahramankazan Vocational School , Program of Electronic Technology, Başkent University,Ankara, Turkey, rilbey@baskent.edu.tr

ABSTRACT

The important new discoveries decades were focussed on the q-calculus based on deformation of quantum spaces. The q-differential operators having the so-called

q -classical polynomials as eigenfunctions fulfils many interesting properties. This is based on orthogonal polynomial whose sequences of the corresponding q-derivatives or Jackson-derivative is also orthogonal. The (p,q) -deformed differential calculus plays an important role in physics and mathematics and its applications and their differential calculus can be given.

The (p,q)- integer is

$$[n]_{p,q} := \frac{p^n-q^n}{p-q}; n = 0,1,2,\ldots, 0 < q < p \leq 1 ,$$

the (p,q)- binomial expansion is

$$(x+y)_{p,q}^n := (x+y)(px+qy)(p^2x+q^2y)\ldots(p^{n-1}x+q^{n-1}y),$$

and the (p,q)-binomial coefficients are

$$\begin{bmatrix} n \\ k \end{bmatrix}_{p,q} := \frac{[n]_{p,q}!}{[k]_{p,q}!\,[n-k]_{p,q}!}.$$

More details about (p,q)- calculus can be found in [2].

(p,q) –calculus is used also in approximation theory. For instance,
In [4] Mursaleen et al. defined the (p,q) – analogue of Bernstein operators as

$$B_{n,p,q}(f;x)=\sum_{k=0}^{n}\begin{bmatrix} n \\ k \end{bmatrix}_{p,q} x^k \prod_{s=0}^{n-k-1}(p^s-q^s x) f\left(\frac{[k]_{p,q}}{[n]_{p,q}}\right), x\in[0,1]$$

and they studied its Korovkin's type approximation properties.

Mittag-Leffler type operators introduced by Özarslan and studied their A-statistical approximation properties in [5] and q-Mittag-Leffler type operators and its approximation properties were obtained in [1].

In this presentation, we introduce the new type of Mittag-Leffler operators in another saying (p,q) - Mittag-Leffler operators by generalizing the q-Mittag-Leffler operators in [1] and then we obtain the approximation properties based on Korovkin's type approximation theorem of (p,q) - Mittag-Leffler operators and establish some direct theorems.

Key Words: Korovkin's type approximation, (p,q)-binomial, (p,q)-Mittag-Leffler operators

REFERENCES

[1] G. İçöz, B. Çekim, q-Analogue of Mittag-Leffler Operators, Miscolc Mathematical Notes, (1) (2017), 211-221

[2] M. N. Hounkonnou, J. Désiré, B. Kyemba, , R(p,q) calculus: differentiation and integration, SUT J. Math. 49 (2) (2013), 145-167.

[3] P.P. Korovkin, Linear Operators and Approximation Theory, Hindustan Publishing Corporation, Delhi, 1960.

[4] M. Mursaleen, K. J. Ansari, A. Khan, On (p, q)-analogue of Bernstein operators, Applied Mathematics and Computation, *266* (2015), 874-882.

[5] M. A. Özarslan A-statistical convergence of Mittag-Leffler operators, Miskolc Math. Notes 14(1) (2013), 209-217.

A new 2-norm derived by bounded linear functionals on the space of p-summable sequences of real numbers

Şükran Konca[1] **and Mochammad Idris**[2]

1 Izmir Bakircay University, Faculty of Engineering and Architecture, Department of Fundamental Sciences, 35665, Izmir, Turkey.
2 Universitas Lambung Mangkurat, Faculty of Mathematics and Natural Sciences, Department of Mathematics, Banjarbaru-Kalimantan Selatan, 70714, Indonesia.

sukran.konca@bakircay.edu.tr, moch.idris@ulm.ac.id

ABSTRACT

The space of p-summable sequences of real numbers can be equipped with norm or n-norm with $n \geq 2$. In this work, we define a new 2-norm which is derived by bounded linear functionals on the space of p-summable sequences of real numbers. We also investigate its relationship among the other 2-norms which are defined by Gähler and Gunawan on this space. We also derive an alternative norm from this new 2-norm and explore its relation with the usual norm on the space of p-summable sequences of real numbers. Alternatively, we redefine another new 2-norm on the space of p-summable sequences of real numbers and investigate its relationship among the Gähler's 2-norm [Lineare 2-normierte Räume, Mathematische Nachrichten] and Gunawan's 2-norm [The space of p-summable sequences and its natural n-norm, Bulletin of the Australian Mathematical Society]. We see that one of these 2-norms is equivalent to the natural 2-norm on this space and there is an equivalence relation among Gähler's 2-norm and Gunawan's 2-norm while the second one is not equivalent to the natural 2-norm on this space and there is no any equivalence relation among the other 2-norms which have already been defined on this space.

Key Words: Bounded linear functionals, the space of p-summable sequences equivalence of norms, 2-norm.

REFERENCES

[1] S. Gähler, Lineare 2-normierte räume, Math. Nachr., 28, (1963), 1-43.

[2] H. Gunawan, The space of p-summable sequences and its natural n-norm, Bull. Austral. Math. Soc., 64, (2001), 137-147.

Alternative criteria for boundedness of one class of matrix operators in weighted sequence spaces

A.Temirkhanova[1], **A. Kalybay**[2]
1 L.N. Gumilyov Eurasian National University, Nur-Sultan, Kazakhstan,
ainura-t@yandex.kz
2 KIMEP University, Almaty, Kazakhstan,
kalybay@kimep.kz

ABSTRACT

Let $1 < p, q < \infty$ and $u = \{u_i\}_{i=1}^{\infty}$, $v = \{v_i\}_{i=1}^{\infty}$ be positive sequences of real numbers. Let $(a_{i,j})$ be a non-negative triangular matrix with entries $a_{i,j} \geq 0$ when $i \geq j \geq 1$ and $a_{i,j} = 0$ when $i < j$. Let $l_{p,v}$ denote the space of sequences of real numbers $f = \{f_i\}_{i=1}^{\infty}$ such that $\|f\|_{p,v} = (\sum_{i=1}^{\infty} |v_i f_i|^p)^{\frac{1}{p}}$, $1 \leq p < \infty$.

We will consider inequality of the following form

$$\|Af\|_{q,u} \leq C\|f\|_{p,v}, \quad \forall f \in l_{p,v} \tag{1}$$

for matrix operator

$$(Af)_i = \sum_{j=1}^{i} a_{i,j} f_j, \quad i \in N. \tag{2}$$

In paper [1] R. Oinarov and Zh. Taspaganbetova introduced an extended class of matrices $O_n^{\pm}$, $n > 0$ and obtained a criterion for the fulfillment of inequality (1) for matrix operators from these classes. The class O_1^{+} consists matrix $(a_{i,j})$ satisfying the following condition: $a_{i,j} \geq 0$ and there exist $d > 1$, a sequence of positive numbers $\{w_i\}_{i=1}^{\infty}$ and a non-negative matrix $(b_{i,j})$, where $b_{i,j}$ is non-decreasing in i and non-increasing in j such that

$$\frac{1}{d}\left(b_{i,k} w_j + a_{k,j}\right) \leq a_{i,j} \leq d\left(b_{i,k} w_j + a_{k,j}\right) \tag{3}$$

for all $i \geq k \geq j \geq 1$.

The class O_1^{-} consists matrix $(a_{i,j})$ satisfying the following condition: $a_{i,j} \geq 0$ and there exist $d > 1$, a sequence of positive numbers $\{w_i\}_{i=1}^{\infty}$ and a non-negative matrix $(b_{i,j})$, where $b_{i,j}$ is non-decreasing in i and non-increasing in j such that

$$\frac{1}{d}\left(a_{i,k} + b_{k,j} w_i\right) \leq a_{i,j} \leq d\left(a_{i,k} + b_{k,j} w_i\right) \tag{4}$$

for all $i \geq k \geq j \geq 1$. Note that first results of the boundedness of matrix operator (2), belonging to the classes O_1^+ and O_1^-, were obtained in [2], [3], respectively.

Therefore, the main goal of this work is to establish new alternative criteria for the fulfilment of the inequality (1) for matrix operators belonging to the classes O_1^+ and O_1^-.

Theorem 1. Let $1 < p \leq q < \infty$ and matrix $(a_{i,j})$ of operator (2) belong to the classes O_1^+. Then inequality (1) holds if and only if $B^+ = \max\{B_1^+, B_2^+\} < \infty$, where

$$B_1^+ = \sup_{k\geq 1}\left(\sum_{n=k}^{\infty} v_n^{-p'}\left(\sum_{i=n}^{\infty} a_{i,n} b_{i,n}^{q-1} u_i^q\right)^{p'}\right)^{\frac{1}{p'}}\left(\sum_{n=k}^{\infty} b_{n,k}^q u_n^q\right)^{-\frac{1}{q'}},$$

$$B_2^+ = \sup_{k\geq 1}\left(\sum_{n=k}^{\infty} v_n^{-p'}\left(\sum_{i=n}^{\infty} a_{i,n} u_i^q\right)^{p'}\right)^{\frac{1}{p'}}\left(\sum_{n=k}^{\infty} u_n^q\right)^{-\frac{1}{q'}}.$$

Moreover $C \approx B^+$, where C is the best constant in (1).

Theorem 2. Let $1 < p \leq q < \infty$ and matrix $(a_{i,j})$ of operator (2) belong to the classes O_1^-. Then inequality (1) holds if and only if $B^- = \max\{B_1^-, B_2^-\} < \infty$, where

$$B_1^- = \sup_{k\geq 1}\left(\sum_{n=1}^{k} u_n^q\left(\sum_{i=n}^{\infty} a_{i,n} v_i^{-p'}\right)^{q}\right)^{\frac{1}{q}}\left(\sum_{n=k}^{\infty} v_n^{-p'}\right)^{-\frac{1}{p}},$$

$$B_2^- = \sup_{k\geq 1}\left(\sum_{n=1}^{k} u_n^q\left(\sum_{i=1}^{n} a_{n,i} b_{n,i}^{p'-1} v_i^{-p'}\right)^{q}\right)^{\frac{1}{q}}\left(\sum_{i=1}^{k} b_{k,i}^{p'} v_i^{-p'}\right)^{-\frac{1}{p}}.$$

Moreover $C \approx B^-$, where C is the best constant in (1).

Key Words: Boundedness, matrix operator, weighted inequality.

REFERENCES

[1] R. Oinarov and Z. Taspaganbetova, Criteria of boundedness and compactness of a class of matrix operators, Journal of Inequalities and Applications, no. 53 (2012), 1-18.

[2] R. Oinarov, L-E. Persson and A. Temirkhanova, Weighted inequalities for a class of matrix operators: the case $p \leq q$. Math. Inequal. Appl. 12 (2009), no. 4, 891-903.

[3] Zh. Taspaganbetova and A. Temirkhanova, Boundedness and compactness criteria of a certain class of matrix operators. Mathematical J. Almaty, 11 (2011), no. 2, 73-85.

Refinement of Berezin Radius Inequalities with Kantorovich Ratio

Hamdullah Başaran[1] and Mehmet Gürdal[2]
1 Department of Mathematics, Süleyman Demirel University, 32260, Isparta,Turkey
07hamdullahbasaran@gmail.com
2 Department of Mathematics, Süleyman Demirel University, 32260, Isparta,Turkey
gurdalmehmet@sdu.edu.tr

ABSTRACT

Linear operators induced by functions are frequently encountered in functional analysis; they include Hankel operators, composition operators, and Toeplitz operators. The inducing function is sometimes referred to as the symbol of the resultant operator. In many circumstances, a linear operator on a Hilbert space $\mathcal{H}$ also gives rise to a function on a subset of a topological space. Hence, we frequently examine operators induced by functions, and we may similarly research functions induced by operators. The Berezin symbol is an outstanding exemplar of an operator-function link. F. Berezin proposed the Berezin transform in [2] and it has proven to be a fundamental tool in operator theory, since many essential features of significant operators are contained in their Berezin transforms. Many researchers in mathematics and mathematical physics are interested in the Berezin symbol of an operator defined on the reproducing kernel Hilbert space. In this context, several mathematicians have conducted substantial research on the Berezin radius inequality. We construct Berezin radius inequalities for Hilbert space operators using the notion of Kantorovich ratio and increasing convex function in this study. Also, we present new results related to Berezin radius inequality. We give the Berezin radius inequality by using operator version of improvement of arithmetic-geometric mean inequality.

Key Words: Berezin symbol, Berezin number, Kantorovich ratio, Young inequality, convex function

REFERENCES

[1] N. Aronzajn, Theory of reproducing kernels, Trans. Amer. Math. Soc., 68 (1950); 337-404.

[2] F.A. Berezin, Covariant and contravariant symbols for operators, Math. USSR-Izvestiya, 6 (1972), 1117-1151.

[3] M.B. Huban, H. Başaran and M. Gürdal, New upper bounds related to the Berezin number inequalities, J. Inequal. Spec. Funct., 12(3) (2021), 1-12.

[4] M.B. Huban, H. Başaran and M. Gürdal, Some new inequalities via Berezin numbers, Turk. J. Math. Comput. Sci., 14(1) (2022), 129-137.

[5] H. Zou, G. Shi, M. Fujii, Refined Young Inequality with Kantorovich constant (Banach space theory and related topics, Rims Kokyuroku, 1735 (2011), 29-34.

From Cesàro and Copson to Morrey

Amiran Gogatishvili [1], **Tuğçe Ünver Yıldız** [2]
1 Institute of Mathematics of the Czech Academy of Sciences Prague, Czech Republic
E-mail: gogatish@math.cas.cz
2 Department of Mathematics, Kirikkale University, Kirikkale, Turkey
E-mail: tugceunver@kku.edu.tr

ABSTRACT

The boundedness of classical operators in local Morrey type spaces have been intensively studied. For the boundedness of the maximal operator, the fractional maximal operator, the Riesz potential, and the singular integral operator in these spaces, there are already certain necessary and sufficient conditions. The main tool in the literature for studying the boundedness of the above operators in local Morrey type spaces is the reduction of these problems to the boundedness of the Hardy operator in weighted Lebesgue spaces on the cone of non-negative, non-increasing functions using the Hölder's inequality. Since the Hölder's inequality is strict, it is possible to obtain better results for the boundedness problem in local Morrey-type spaces by using the characterization of embeddings between these spaces.

The aim of this talk is to give the characterization of embeddings between weighted local Morrey type spaces and weighted complementary local Morrey-type spaces. The characterization is known only for the range of parameters where the duality argument works. However, we will use a different approach and show that certain n-dimensional inequalities are equivalent to inequalities in the 1-dimensional setting. Thus, the recent characterization of embeddings between Copson and Cesàro spaces will allow us to study the boundedness of the identity operator between weighted complementary Morrey-type local spaces and weighted Morrey-type local spaces.

Key Words: Local Morrey-type spaces, Cesàro spaces, Copson spaces, embeddings, reduction.

REFERENCES

[1] Gogatishvili and T. Ünver, Embeddings between weighted local Morrey-type spaces, Nonlinear Stud. 26 no: 4 (2019), 805–819.

[2] A. Gogatishvili, R. Mustafayev and T. Ünver, Embeddings between weighted Copson and Cesàro function spaces. Czechoslovak Math. J. 67(142) no. 4 (2017), 1105–1132.

[3] A. Gogatishvili, L. Pick and T. Ünver, Weighted inequalities involving Hardy and Copson operators, (2022), arXiv:2203.00596 [math.FA].

A New Accelerated Fixed-Point Algorithm with Application to Image Restoration Problem

Ebru ALTIPARMAK [1] **and Ibrahim KARAHAN** [2]
1 Erzurum Technical University, ebru.altiparmak@erzurum.edu.tr
2 Erzurum Technical University, ibrahimkarahan@erzurum.edu.tr

ABSTRACT

Fixed point theory is an important topic that has many applications in various fields of mathematics. One of the application areas of fixed-point theory is image restoration problems. These problems are important issues in image processing. Image restoration is a classical inverse problem that has been extensively studied in various applications such as medical imaging, astronomical imaging, remote sensing, and video or image coding. The image restoration problem can be formulated by the inversion of the following way:

$z = Ax + y,$

where $x \in \mathbb{R}^{n\times 1}$ is the original image, $z \in \mathbb{R}^{m\times 1}$ is the degenerated image, y is the additive noise and $A \in \mathbb{R}^{m\times n}$ is the blur function. Image restoration is the process of obtaining a relatively clear image from a distorted or imaged image. So, the aim of image restoration is to increase the quality of the images.

In this study, we present a new fixed-point algorithm for finding a common fixed point of an infinite family of nonexpansive mappings in the Hilbert space. Moreover, the strong convergence theorem is established under some suitable conditions. We give an application of our algorithm for the monotone inclusion problem, the convex minimization problem, and also the image restoration problem. Finally, we conclude that our algorithm outperforms better than some other algorithms in image restoration.

Key Words: Monotone inclusion problem, convex minimization problem, image restoration problem.

REFERENCES

[1] H.H. Bauschke and P.L. Combettes, Convex Analysis and Monotone Operator Theory in Hilbert Space, CMS Books in Mathematics Springer, New York, 2011.

[2] J. Puangpee and S. Suantai, A New Accelerated Viscosity Iterative Method for an Infinite Family of Nonexpansive Mappings with Applications to Image Restoration Problems, Mathematics, 8(4), (2020):615.

[3] P.L. Lions and B. Mercier, Splitting Algorithms for the sum of two nonlinear operators, SIAM Journal on Numerical Analysis, 16(6), (1979), 964-979.

Two-sided estimates of the norm of an operator with a logarithmic singularity for *p > q*

A.M. Abylayeva[1], R. Oinarov[2], B.N. Seilbekov[3]

1 L.N. Gumilyov Eurasian National University, Nur-Sultan, Kazakhstan

abylayeva_b@mail.ru, o_ryskul@mail.ru, bolat_3084@mail.ru

ABSTRACT

Let $I = (0,\infty)$ and the functions v, u are positive almost everywhere, locally integrable on the interval I.

Let $1 < p,q < \infty$ and $p' = \frac{p}{p-1}$. Let us denote by $L_{q,v} \equiv L_q(v,I)$ the set of functions f measurable on I for which

$$\|f\|_{q,v} = \left(\int_0^\infty |f(x)|^q v(x) dx \right)^{\frac{1}{q}} < \infty .$$

Let W is a positive, strictly increasing and locally absolutely continuous function on the interval I. Assume $\frac{dW(x)}{dx} = w(x)$, for almost all $x \in I$.

Consider the operator

$$T_{\alpha,\beta} f(x) = \int_0^x \frac{\left(ln \frac{W(x)}{W(x)-W(s)} \right)^\beta}{(W(x)-W(s))^{1-\alpha}} u(s) f(s) dW(s), \quad x \in I, \tag{1}$$

where $\alpha > 0$, $\beta \geq 0$.

When $\beta = 0$ the operator $T_{\alpha,\beta}$ is called the operator of fractional integration of the function f with respect to the function W when $u \equiv 1$ ([1], p.248).

The main purpose of the paper is to establish the boundedness and compactness of the operator (1) $T_{\alpha,\beta}$ from $L_{p,w}$ to $L_{q,v}$ with the following ratios of parameters $0 < q < p < \infty$.

THEOREM 1. Let $0 < \alpha < 1$, $0 < q < p < \infty$, $p > \frac{1}{\alpha}$ and $\beta \geq 0$. Let the function u be nonincreasing on I. Then the operator $T_{\alpha,\beta}$ is bounded from $L_{p,w}$ to $L_{q,v}$ if and only if

$$B_{\alpha,\beta} = \left(\int_0^\infty \left(\int_0^z W^{p'\beta}(s) u^{p'}(s) w(s) ds \right)^{\frac{q(p-1)}{p-q}} \times \right.$$

$$\left. \times \left(\int_z^\infty v(x) W^{q(\alpha-\beta-1)}(x) dx \right)^{\frac{q}{p-q}} W^{q(\alpha-\beta-1)}(z) v(z) dz \right)^{\frac{p-q}{pq}} < \infty$$

,

and $T_{\alpha,\beta} \approx B_{\alpha,\beta}$, where $T_{\alpha,\beta}$ is the norm of the operator $T_{\alpha,\beta}$ from $L_{p,w}$ to $L_{q,v}$.

THEOREM 2. Let $\frac{1}{p} < \alpha < 1$, $1 < q < p < \infty$ and $\beta \geq 0$. Let the function u be nonincreasing on I. Then the operator $T_{\alpha,\beta}$ is compact from $L_{p,w}$ to $L_{q,v}$ if and only if $B_{\alpha,\beta} < \infty$.

REFERENCES

[1] Samko S.G, Kilbas A.A., Marichev O.I.: Integrals and Derivatives of Fractional Order and Some of Their Applications. Science and Technology, Minsk (1987) (in Russian)

On Ideals $\mathfrak{I}_c^{(q)}$

Mustafa Gülfırat [1], **Nilay Şahin Bayram**[2]
1 Department of Mathematics, Faculty of Science, Ankara University, Ankara, Türkiye
2 Department of Electrical and Electronics Engineering, Faculty of Engineering, Başkent University, Ankara, Türkiye

ABSTRACT

In the papers [3] and [4] the notion of $\mathfrak{I}$ -convergence of sequences of real numbers is introduced and its basic properties are investigated. The $\mathfrak{I}$ -convergence generalizes the notion of the statistical convergence and it is based on the ideal $\mathfrak{I}$ of subsets of the set $\mathbb{N}$ of positive integers. They presented some examples of admissible ideals.

In the present work we mainly deal with ideals $\mathfrak{I}_c^{(q)}$. For any $q \in (0,1]$ the set

$$\mathfrak{I}_c^{(q)} = \left\{ E \subseteq \mathbb{N}: \sum_{i \in E} \frac{1}{i^q} < \infty \right\}$$

is an admissible ideal. Recall that $\mathfrak{I}_c^{(q)} \subset \mathfrak{I}_d$ (see e.g., [2]), where the ideal $\mathfrak{I}_d$ is the class of all subsets of positive integers that has asymptotic density zero. In 2011 Gogolo et al. studied the properties of ideals $\mathfrak{I}_c^{(q)}-$ related to the notion of $\mathfrak{I}-$convergence and they showed that $\mathfrak{I}_c^{(q)}-$ and $\mathfrak{I}_c^{(q)*}-$convergences are equivalent ([2]). They also ([2]) introduced the class $\mathcal{T}_q$ of lower triangular nonnegative matrices as follows:

A matrix $T = (t_{nk})$ belongs to the class $\mathcal{T}_q$ if and only if it satisfies the following conditions:

(I) $lim_{n\to\infty} \sum_{k=1}^{n} t_{nk} = 1$

(q) $E \subset \mathbb{N}$ and $E \in \mathfrak{I}_c^{(q)}$, then $lim_{n\to\infty} \sum_{k \in E} t_{nk} = 0$, $q \in (0,1]$.

By $c^{\mathfrak{I}(q)}$, $c^{\mathfrak{I}(q)}(b)$ we denote the set of all $\mathfrak{I}_c^{(q)}-$ convergent sequences, the set of all bounded $\mathfrak{I}_c^{(q)}-$ convergent sequences, respectively.

We study results motivated by those of [1] and [2]. In particular we study bounded multipliers of bounded $\mathfrak{I}_c^{(q)}-$convergent sequences, and we show that

$$M\left(c^{\mathfrak{I}(q)}(b)\right) = \bigcap_{T \in \mathcal{T}_q} M(c_T(b)).$$

Key Words: Ideal, Ideal convergence, Multipliers.

REFERENCES

[1] K. Demirci, C. Orhan, Bounded multipliers of bounded A−statistically convergent sequences. Journal of Mathematical Analysis and Applications 235 (1999),122-129.

[2] J. Gogola, M. Mačaj, T. Visnyai, On $\mathfrak{I}_c^{(q)}$ -convergence. Annales Mathematicae et Informaticae 38 (2011),27-36.

[3] P. Kostyrko, T. Šalát, W. Wilczyński $\mathfrak{I}$ -convergence. Real Analysis Exchange 26(2) (2000),669-686.

[4] P. Kostyrko, M. Macaj, T. Šalát, M. Sleziak, $\mathfrak{I}$ -convergence and extremal I-limit poits, Mathematica Slovaca 55(4) (2005),443–464.

[5] T. Šalát, B.C. Tripathy, M. Ziman, On some properties of $\mathfrak{I}$ –convergence, Tatra Mountains Mathematical Publications 28 part II (2004),279-286.

[6] R.C. Walker, The Stone-Čech Compactification, New York, NY, USA: Springer-Verlag, 1974.

Some Results on Commutators

Müjdat Ağcayazı [1]
1 Aydin Adnan Menderes University Faculty of Education Department of Mathematics and Science Education
mujdat.agcayazi@adu.edu.tr

ABSTRACT

One of the most important issues in harmonic and functional analysis is the boundedness properties of the operators in many function spaces. In this context, the operators we will work with will be maximal commutators, commutators of the Hardy−Littlewood maximal operators M and commutators of the Fefferman−Stein sharp maximal operators $M^{\#}$. As it is known, these operators are used in almost all areas of mathematical analysis and the most important results are obtained by using these operators.

Let b be a locally integrable function on $\mathbb{R}^n$. We define the commutator operators $[M,b]f$ and $[M^{\#},b]f$ as

$$[M,b]f := M(bf) - bMf,$$

and

$$[M^{\#},b]f := M^{\#}(bf) - bM^{\#}(f),$$

respectively.

M. Milman and T. Schonbek [1] proved that commutator is a bounded map from Lebesgue spaces $L_p(\mathbb{R}^n)$ onto itself, $p > 1$, when the nonnegative symbol belongs to BMO ($\mathbb{R}^n$).

The necessary and sufficient conditions for the boundedness of $[M,b]$ and $[M^{\#},b]$ on Lebesgue spaces $L_p(\mathbb{R}^n)$ was shown by J. Bastero, M. Milman and F. J. Ruiz in [2].

P. Zhang and J. Wu [3] extended the results in [1] and [2] to Lebesgue spaces with variable exponent $L_{p(\cdot)}(\mathbb{R}^n)$.

In this talk, the above-mentioned results will be obtained in more general Banach function spaces, and necessary and sufficient conditions for the boundedness of operators will be obtained. Finally, some applications will be presented.

Key Words: Commutators, Maximal operators, BMO.

REFERENCES

[1] J. Bastero, M. Milman, and F. J. Ruiz, Commutators for the maximal and sharp functions, Proc. Amer. Math. Soc. 128 (2000), 3329–3334.

[2] M. Milman and T. Schonbek, Second order estimates in interpolation theory and applications, Proc. Amer. Math. Soc. 110 (1990), 961–969.

[3] P. Zhang and J. Wu, Commutators for the maximal functions on Lebesgue spaces with variable exponent, Math. Inequal. Appl. 17 (2014), 1375–1386.

Spanne and Adams type Results in the Vanishing Generalized Weighted Local Morrey Spaces

Abdulhamit Kucukaslan[1]

1. Ankara Yıldırım Beyazıt University, Faculty of Aerospace Sciences, Department of Aerspace Engineering, 06010, Ankara, Turkey a.kucukaslan@aybu.edu.tr

ABSTRACT

Morrey spaces were introduced by Morrey in [2]. These spaces appeared to be useful in the study of local behavior properties of the solutions of second order elliptic PDEs. The vanishing Morrey space of the classical Morrey spaces was introduced by Vitanza in [4] and applied there to obtain a regularity result for elliptic PDEs. Later in [5] Vitanza proved an existence theorem for a Dirichlet problem, under weaker assumptions then those introduced by Miranda in [1] and a regularity result assuming that the partial derivatives of the coefficients of the highest and lower order terms belong to vanishing Morrey spaces depending on the dimension.

The vanishing generalized Morrey space and vanishing generalized local Morrey space was introduced by Samko in [3]. The boundedness of the multi-dimensional Hardy type operators, maximal, potential and singular operators in these spaces were proved in [3].

The generalized fractional maximal operators M_ρ was initially investigated in by Nakai in 1994. Nakai introduced the generalized Morrey spaces and proved the boundedness of the generalized fractional integral operator in these spaces. Nowadays many authors have been culminating important observations about the generalized fractional maximal operators M_ρ especially in connection with Morrey-type spaces.

In this talk, we prove the Spanne-type and Adams-type boundedness of the generalized fractional maximal operator M_ρ from the vanishing generalized weighted local Morrey spaces $VLM_{p,\varphi_1}(\omega^p)$ to another vanishing generalized weighted local Morrey spaces $VLM_{q,\varphi_2}(\omega^q)$ with ω^q, and from the vanishing generalized weighted local Morrey spaces $VLM_{1,\varphi_1}(\omega \quad)$ to the vanishing generalized weighted weak local

Morrey spaces $VWLM_{q,\varphi_2}(\omega^q)$ with ω ; and $VM_{p,\varphi^{\frac{1}{p}}}(\omega)$ to the vanishing generalized weighted Morrey spaces $VM_{q,\varphi^{\frac{1}{q}}}(\omega)$ with the weight function ω , and from the vanishing generalized weighted Morrey spaces $VM_{1,\varphi}(\omega)$ to the vanishing generalized weighted weak Morrey spaces $VWM_{q,\varphi^{\frac{1}{q}}}(\omega)$, respectively Also, we get a new theorem for the Spanne-type result of the generalized fractional maximal operator M_ρ from the vanishing generalized local Morrey spaces VLM_{p,φ_1} to vanishing generalized local Morrey spaces VLM_{q,φ_2}, including weak estimates. The all weight functions belong to Muckenhoupt-Weeden classes $A_{p,q}$.

Key Words: Generalized fractional maximal operator, Vanishing generalized weighted local Morrey space, Vanishing generalized Morrey space, Muckenhoupt-Weeden classes.

REFERENCES

[1] C. Miranda, Sulle equazioni ellittiche del secondo ordine di tipo non variazionale, a coefficienti discontinui, Ann. Math. Pura E Appl., 63(4)(1963), 353-386.

[2] C. B. Morrey, On the solutions of quasi-linear elliptic partial differential equations, Trans. Amer. Math. Soc., 43(1938), 126-166.

[3] N. Samko, Weighted Hardy operators in the local generalized vanishing Morrey spaces, Positivity 17(2013), 683-706.

[4] C. Vitanza, Functions with vanishing Morrey norm and elliptic partial differential equations, Proceedings of Methods of Real Analysis and Partial Differential Equations, Capri, Springer (1990), 147-150.

[5] C. Vitanza, Regularity results for a class of elliptic equations with coefficients in Morrey spaces, Ricerche di Matematica 42(2)(1993), 265-281.

On Some Properties of Double Series Spaces Derived by Cesàro Means

Okan BODUR [1], Canan HAZAR GÜLEÇ [2]
1 Pamukkale University, Faculty of Science, okan.bodur@hotmail.com
2 Pamukkale University, Faculty of Science, gchazar@pau.edu.tr

ABSTRACT

There has been a lot of interest recently in the generalizations of single sequence spaces to double sequence spaces. The initial works on double sequences have been given by Bromwich [2] and Pringsheim [6]. Later on, double sequences and double series have been studied by Móricz [4], Móricz and Rhoades [5], and many others. Also, Başar and Sever have introduced the Banach space $\mathcal{L}_k$ of double sequences corresponding to the well known classical sequence space ℓ_k of single sequences [1]. Furthermore, for the special case $k = 1$, the space $\mathcal{L}_k$ is reduced to the space $\mathcal{L}_u$, which was introduced by Zeltser [8]. More recently, some important results on single sequence spaces have been extended to double sequence spaces by means of matrix domain of four dimensional matrices [3,7]. In this study, our main purpose is to investigate some topological and algebraic properties of the absolutely double series space $|C_{1,1}|_k$, which is defined by combining the first order Cesàro means with the concept of absolute summability method for $k \geq 1$. To explain in more detail, we show that the set $|C_{1,1}|_k$ becomes a linear space with the coordinatewise addition and scalar multiplication, and also $|C_{1,1}|_k$ is a Banach space. Moreover, we obtain that the double series space $|C_{1,1}|_k$ is linearly norm isomorphic to the space $\mathcal{L}_k$ for $1 \leq k < \infty$.

Key Words: Double Series, Cesàro Means, Absolute Summability Method.

REFERENCES

[1] Başar, F. and Sever, Y., The space $\mathcal{L}_k$ of double sequences, Math. J. Okayama Univ., 51, (2009), 149-157.

[2] Bromwich, T. J., An Introduction to the Theory of Infinite Series, Macmillan, New York, NY, USA, (1965).

[3] Mursaleen, M. and Başar, F., Domain of Cesàro mean of order one in some spaces of double sequences, Studia Scientiarum Mathematicarum Hungarica, 51, (2014), 335-356.

[4] Móricz, F.,Extensions of the spaces c and c_0 from single to double sequences, Acta Math. Hungar., 57, (1991), 129-136.

[5] Móricz, F. and Rhoades, B. E., Almost convergence of double sequences and strong regularity of summability matrices, Mathematical Proceedings of the Cambridge Philosophical Society, 104 (2), (1988), 283-294.

[6] Pringsheim, A., Zur Theorie der zweifach unendlichen Zahlenfolgen, Mathematische Annalen, 53(3), (1900), 289-321.

[7] Sarıgöl, M. A., On equivalence of absolute double weighted mean methods, Quaestiones Mathematicae, 44(6), (2021), 755-764.

[8] Zeltser, M, On conservative matrix methods for double sequence spaces, Acta Math. Hung., 95(3), (2002), 225-242.

Two-weighted inequalities for Riesz potential and its commutators in generalized weighted Morrey spaces

Canay Aykol[1] and Javanshir J. Hasanov[2]
1 Department of Mathematics, Ankara University, Ankara, Turkey aykol@science.ankara.edu.tr
2 Azerbaijan State Oil and Industry University, Baku, Azerbaijan
hasanovjavanshir@gmail.com

ABSTRACT

In this talk we find the conditions for the boundedness of Riesz potential I^{α} and its commutators from the generalized weighted Morrey spaces $\mathcal{M}_{\omega_1}{}^{p,\varphi_1}(\mathbb{R}^n)$ to the generalized weighted Morrey spaces $\mathcal{M}_{\omega_2}{}^{p,\varphi_2}(\mathbb{R}^n)$, where $0 < \alpha < n$, $1 < p < \frac{n}{\alpha}$, $\frac{1}{p} - \frac{1}{q} = \frac{\alpha}{n}$, φ_1, φ_2 positive measurable functions on $\mathbb{R}^n x(0,\infty)$, $(\omega_1, \omega_2) \in A_{p,q}(\mathbb{R}^n)$ and $b \in \mathrm{BMO}(\mathbb{R}^n)$. Furthermore, we give some applications of our results.

Key Words: Maximal operator, Riesz potential, commutator, weighted Lebegue space, generalized weighted Morrey space, BMO space.

REFERENCES

[1] V. Burenkov, A. Gogatishvili, V.S. Guliyev and R. Mustafayev, Boundedness of the fractional maximal operator in local Morrey-type spaces, Complex Var. Elliptic Equ. 55 (8- 10) (2010), 739-758.

[2] V.S. Guliyev, Generalized weighted Morrey spaces and higher order commutators of sublinear operators, Eurasian Math. J. 3 (3) (2012), 33- 61.

[3] V.S. Guliyev, Generalized local Morrey spaces and fractional integral operators with rough kernel, J. Math. Sci. (N. Y.) 193 (2) (2013), 211-227.

[4] V.S. Guliyev, T. Karaman, R.Ch. Mustafayev and A. Serbetci, Commutators of sublinear operators generated by Calderon-Zygmund operator on generalized weighted Morrey ´ spaces, Czechoslovak Math. J. 64 (139) (2) (2014), 365-386.

[5] T. Karaman, V.S. Guliyev and A. Serbetci, Boundedness of sublinear operators generated by Calderon-Zygmund operators on generalized weighted Morrey spaces ´ , An. Stiint. Univ. Al. I. Cuza Iasi. Mat. (N. S.) 60 (1) (2014), 227-244.

[6] V. Kokilashvili and M. Gabidzashvili, Two- weight weak type inequalities for fractional type integrals, Preprint. Math. Inst. Czech. Acad. Sci. Prague 45 (1989).

[7] V. Kokilashvili and A. Meskhi, Two-weight inequalities for fractional maximal functions and singular integrals in $L^{p(\cdot)}$ spaces, J. Math. Sci. (N.Y.) 173 (6) (2011), 656-673.

[8] N. Samko, Weighted Hardy and singular operators in Morrey spaces, J. Math. Anal. Appl. 350 (1) (2009), 56-72.

Inequalities on Riemannian Warped Product Submersions for Vertical Casorati Curvatures

Irem Kupeli Erken[1], Cengizhan Murathan[2] and Aliya Naaz Siddiqui[3]

1 Bursa Technical University, Faculty of Engineering and Natural Sciences, Department of Mathematics, Bursa, TURKEY.
E-mail address: irem.erken@btu.edu.tr
2 Bursa Uludag University, Art and Science Faculty, Department of Mathematics, 16059 Bursa, TURKEY.
E-mail address: cengiz@uludag.edu.tr
3 Department of Mathematics, Maharishi Markandeshwar (Deemed to be University), Mullana, 133207, Ambala-Haryana, India.
E-mail address: aliyanaazsiddiqui9@mmumullana.org

ABSTRACT

The first and second authors introduced Riemannian warped product submersions and discussed interesting fundamental geometric properties of such submersions in [1]. In the present paper, we extend this study to put light on the curvature properties of such submersions and then obtain optimal inequalities for Riemannian warped product submersions involving vertical Casorati curvatures. Also, we discuss under which conditions, the equality case of inequality can hold with an example.

Key Words: Riemannian immersion, Riemannian submersion, Warped product, Vertical Casorati curvatures, Einstein manifold.

REFERENCES

[1] I.K. Erken, C. Murathan, Riemannian Warped Product Submersions, Results in Mathematics, 76(1) (2021) DOI: 10.1007/s00025-020-01310-4.

[2] A.N. Siddiqui, M.D. Siddiqi, A.H. Alkhaldi and A. Ali, Lower Bounds on Statistical Submersions with vertical Casorati curvatures, Intern. J. Geom. Meth. Mod. Phys., (2022) https://doi.org/10.1142/S021988782250044X.

[3] C.W. Lee, J.W. Lee, B. Sahin and G.E. Vilcu, Optimal inequalities for Riemannian maps and Riemannian submersions involving Casorati curvatures, Annali di Matematica Pura ed Applicata, 200(3) (2020) 1-19 https://doi.org/10.1007/s10231-020-01037-7.

APPLIED MATHEMATICS

Singularly Perturbed Two-Point Fuzzy Boundary Value Problems

Selami BAYEĞ[1], Nurettin DOĞAN*[2], Raziye MERT[3], Ömer AKIN[4]

[1]Turkish Aeronautical Association University, Engineering Faculty, Department of Industrial Engineering, Ankara Turkey
[2]Selçuk University, Technology Faculty, Computer Engineering Department, Alaaddin Keykubat Campus
Konya Turkey
[3]Adana Alparslan Türkeş Science and Technology University, Engineering Faculty, Software Engineering
Department, Adana Turkey
[4] TOBB University of Economics and Technology, Faculty of Science and Arts, Department of Mathematics, Ankara, Turkey

ABSTRACT

In this work, we have firstly introduced singularly perturbed two-point fuzzy boundary value problems (SPTPFBVP) and then we have given an algorithm for obtaining the solutions of them by using Zadeh's extension principle given. We have presented some results on the behaviour of the α-cuts of the solutions. To show the robustness of the given algorithm, we have fuzzified some examples with different boundary layers given in literature and then we have successfully applied the algorithm.

Key Words: Fuzzy Differential Equation, Two Point Boundary Value Problem, Singularly Perturbed, Zadeh's Extension Principle, Mathematics.

REFERENCES

[1] Neyfeh, Perturbation Methods (Wiley, 1973).
[2] R. J. O'Malley, Introduction to singular perturbations (Academic Press, 1974).
[3] R. J. O'Malley, Singular perturbation methods for ordinary differential equations (Springer Verlag, 1991).
[4] M. Alquran, & N. Doğan, "Variational iteration method for solving two parameter singularly perturbed two point boundary value problem", Applications and Applied Mathematics: An International Journal (AAM) 5 (2010) 81–95.
[5] N. Doğan, M. Al Quran, V. S. Ertürk & S. Momani, "Variational iteration method for solving singularly perturbed two point boundary value problems", International Journal of Pure and Applied Mathematics 58 (2010) 11.
[6] N. Doğan, V. S. Ertürk, & Ö. Akın, "Numerical treatment of singularly perturbed two-point boundary value problems by using differential transformation method", Discrete Dynamics in Nature and Society 2012 (2012).
[7] N. Doğan, V. S. Ertürk, & S., Momani, "Application of He' s variational iteration method for solving singularly perturbed volterra integral equations", World Applied Sciences Journal (2013)22 1657–166.

Classification of 3D Point Clouds Using Graph Communities and Deep Learning

Ömer Akgüller [1] and Mehmet Ali Balcı [2]
1 Muğla Sıtkı Koçman University, Faculty of Science, Department of Mathematics,
oakguller@mu.edu.tr
2 Muğla Sıtkı Koçman University, Faculty of Science, Department of Mathematics,
mehmetalibalci@mu.edu.tr

ABSTRACT

One of the most fundamental and adaptable dataset formats are point clouds. It is crucial to work directly with this representation without having to go through the intermediary phase, which might add computational cost and fit-to-face issues, given the expanding popularity and extremely broad uses of this data source. Point clouds are commonly employed in the depiction of high-dimensional manifolds, which is an important topic. Nearly every field, from computational biology to image analysis to financial data, uses such high-dimensional and generic isodimensional data [1,2]. Manifold reconstruction is not possible in this situation due to the high dimensionality; thus, the necessary computations must be made directly on the raw data, or point cloud. It's crucial to use graph representations while analysing point clouds. In this paper, the community similarity measures of graph theory are used to provide a solution to the point cloud classification issue. With the aid of internal similarity matrices discovered by contrasting each community with the others, classification of fixed number communities of synthetic 3D point clouds was carried out using deep learning. In this study, fixed-number community determination approaches and deep learning techniques employing hybrid Long Short Term Memory and Convolution Neural Network architectures are contrasted in terms of classification performance. We present the point cloud classification technique' most effective number of communities and most effective deep learning hyperparameters.

Key Words: Point cloud classification, graph communities, deep learning.

REFERENCES

[1] Q. Wang and M. K. Kim, Applications of 3D point cloud data in the construction industry: A fifteen-year review from 2004 to 2018. Advanced Engineering Informatics, 39, (2019) 306-319.

[2] A. Nguyen and B. Le, 3D point cloud segmentation: A survey. In 2013 6th IEEE conference on robotics, automation and mechatronics (RAM) (2013), 225-230, IEEE

Deep Learning Based Classification of Eigenworms with Gramian Angular Fields

Mehmet Ali Balcı [1] and Ömer Akgüller [2]
1 Muğla Sıtkı Koçman University, Faculty of Science, Department of Mathematics, mehmetalibalci@mu.edu.tr
2 Muğla Sıtkı Koçman University, Faculty of Science, Department of Mathematics, oakguller@mu.edu.tr

ABSTRACT

The roundworm Caenorhabditis elegans is frequently employed as a model organism in genetic research. It is well recognized that understanding the behavioural genetics of these worms' movement might be a helpful indication for studies on soil biology. Several research have used systems to track worm movement on an agar plate and quantify a variety of human-defined parameters [1,2]. Combinations of the six fundamental forms, or eigenworms, can be used to show the range of shapes that Caenorhabditis elegans can take on an agar plate. Once the worm is drawn out, each frame of worm motion may be represented by six scalars, one for each dimension's amplitude, when the figure is projected onto the worm's six cores. This kind of data format matches to time series mathematically. The topic of categorizing individual worms as wild type or mutant based on time series is examined as a time series classification problem in this work utilizing the data gathered for the study reported in [1]. In this study, classification of images derived from time series is used in place of models with traditional time series classifier and deep learning architecture. Deep learning is used to classify Gramm Angular Field images of time series, and comparisons with several models are shown.

Key Words: Mathematical Biology, deep learning, time series classification.

REFERENCES

[1] A. E. Brown, E. I. Yemini, L. J. Grundy, T. Jucikas and W. R. Schafer, A dictionary of behavioral motifs reveals clusters of genes affecting Caenorhabditis elegans locomotion. *Proceedings of the National Academy of Sciences*, 110(2), (2013), 791-796.

[2] E. I. Yemini, T. Jucikas, L. J. Grundy, A. E. Brown and W. R. Schafer, A database of Caenorhabditis elegans behavioral phenotypes. Nature methods, 10(9), (2013), 877-879

Stability Analysis of Prey – Predator System Involving Multiple Allee Effect

Deniz ELMACI [1], **Figen KANGALGİL** [2]
1 Dokuz Eylul University, Bergama Vocational School, deniz.elmaci@deu.edu.tr
2 Dokuz Eylul University, Bergama Vocational School, figen.kangalgil@deu.edu.tr

ABSTRACT

Allee effects have already been demonstrated to play significant roles in the dynamics of many populations. These effects are directly related to the vulnerability of species to extinction and are increasingly receiving recognition from both theoretically oriented and ecologists. Two or more Allee effects may happen simultaneously in the same population, despite this not being commonly known. In this study, a mathematical model is proposed to study a stability analysis of a diffusive prey–predator system with multiple Allee effect. The existence of biological feasible fixed points with their stability properties are determined. Theoretical discussion is illustrated through numerical simulations.

Key Words: Allee effect, prey-predator model, local stability.

REFERENCES

[1] B. Tiwari, S. N. Raw and P. Mishra, Qualitative analysis of a spatiotemporal prey–predator model with multiple Allee effect and schooling behaviour, Nonlinear Dyn. 102 (2020), 3013-3038.

[2] F. Kangalgil, Neimark–Sacker bifurcation and stability analysis of a discrete-time prey–predator model with Allee effect in prey, Advances in Difference Equations, 92 (2019), 1-12.

[3] F. Kangalgil, Ö. Ak Gümüş, Allee effect in a new population model and stability analysis. Gen. Math. Notes (2016).

[4] Q. Din, T. Donchev and D. Kolev, Stability, Bifurcation Analysis and Chaos Control in Chlorine Dioxide-Iodine-Malonic Acid Reaction, MATCH Commun. Math. Comput. Chem. 79 (2018) 577-606.

[5] S. Zhou, Y. Liu, G. Wang, The stability of predator–prey systems subject to the Allee effects. Theor. Popul. Biol. 67 (2005), 23–31.

Oscillation criteria for third-order nonlinear neutral differential equations via comparison principles

Orhan Özdemir[1]
1 Department of Mathematics, Faculty of Arts and Science,
Tokat Gaziosmanpasa University, Tokat, Turkey.
Email:orhanozdemir37@yahoo.com

ABSTRACT

The oscillation and asymptotic behaviour of solutions for different classes of functional differential equations and functional dynamic equations on time scales is an active and important area of research. The rapidly growing interest in qualitative behaviour of solutions of neutral differential equations is motivated by their applications in natural sciences and engineering.

In this talk, we will consider a class of third order nonlinear functional differential equations that involves both delayed and advanced arguments in neutral term. Under certain conditions, we aim to present some comparison theorems that guarantee the oscillation of all solutions of the studied equation. The results obtained are based on comparisons with associated first order delayed differential inequalities and first order delayed differential equations. Let us point out that comparison theorems presented in this study can be applicable to both cases where the neutral coefficients of the differential equation are unbounded and/or bounded. Some illustrative examples are also provided to demonstrate applicability and significance of the main results.

Key Words: Oscillation, comparison theorem, third-order, neutral differential equations.

REFERENCES

[1] O. Özdemir and Ş. Kaya, Comparison theorems on the oscillation of third-order functional differential equations with mixed deviating arguments in neutral term, Differ. Equ. Appl. 14(1) (2022), 17-30.

[2] G. E. Chatzarakis, R. Srinivasan and E. Thandapani, Oscillation results for third-order quasi-linear Emden-Fowler differential equations with unbounded neutral coefficients, Tatra Mt. Math. Publ., 80 (2021), 1-14.

Asymptotics of the eigenvalues of a boundary value problem for the operator Schrödinger equation with a spectral parameter, which is included polynomially in the boundary condition

Ilyas F. Hashimoglu [1]
[1] Karabuk University, Faculty of Business, Department of Business Administration, Turkey
i.hasimoglu@karabuk.edu.tr

ABSTRACT

In this study, on the space $H_1 = L_2(H,[0,1])$, where H is a separable Hilbert space, we study the asymptotics of the eigenvalues of the following boundary value problem for the operator Schrödinger equation:

$$-u''(x) + \frac{\nu^2 - \frac{1}{4}}{x^2} u(x) + Au(x) = \lambda u(x), \qquad x \in (0,1),\ \nu \geq \frac{1}{2} \tag{1.1}$$

$$u(0) = 0, \quad u'(1) + (\alpha_0 + \alpha_1 \lambda + \alpha_2 \lambda^2) u(1) = 0, \tag{1.2}$$

where λ is a spectral parameter; A is a self-adjoint, positive-definite operator in H; the inverse operator A^{-1} is completely continuous in H, and $\alpha_0, \alpha_1, \alpha_2$ are reel numbers: $\alpha_0 > 0$ and $\alpha_2 < 0$.

In $[1]$, an investigation has been done on the asymptotic behavior of the eigenvalues of the following boundary value problem for the operator Schrödinger equation (1.1) with the boundary conditions

$$u(0) = 0, \quad u'(1) - \lambda^2 u(1) = 0 \tag{1.3}$$

It has been proven that the eigenvalues of the problem $(1.1) - (1.2)$ are real and simple. Furthermore, it was shown that the eigenvalues asymptotically behave as $\left(\frac{1}{2}\nu\pi + \frac{3}{4}\pi + \pi n\right)^2 + \mu_k$ or $(j_{n+1})^2 + \mu_k$, where the numbers $j_1 < j_2 < j_3 < \cdots < j_n < \cdots$ are the roots of the equation $J_\nu(z) = 0$, and $\mu_k = \mu_k(A)$ are the eigenvalues of the operator A.

Keywords: Operator differential equations, spectrum, eigenvalue, asymptotic formula, Hilbert space

REFERENCES

[1] F. Hashimoglu, Asymptotics of the eigenvalues of a boundary value problem for the operatorSchrödinger equation with boundary conditions nonlinearly dependent on the spectral parameter, *Journal of Samara State Technical University, Ser. Physical and Mathematical Sciences*, Vol. 25, Issue 4, pp. 607-615, 2021. DOI: 10.14498/vsgtu1894

Analytical Solutions of Conformable Time Fractional Partial Differential Equation

Ali KURT [1], **Abdulsamet BEKTAŞ** [2]
1 Department of Mathematics, Faculty of Science, Pamukkale University
akurt@pau.edu.tr
2 Department of Mathematics, Faculty of Science, Pamukkale University,
s.bektas33@hotmail.com

ABSTRACT

In this study analytical solutions of conformable fractional partial differential equation are obtained by using $\exp(-\emptyset(\varepsilon))$ method. At the beginning chain rule and wave transform are used for changing nonlinear fractional partial differential equation into nonlinear ordinary differential equation where the derivatives are integer order. As a result of this process, there is no need to use either discretization or linearization to reduce the equation. Also 3D graphical representations are given to view the geometrical behaviour of obtained solutions.

Key Words: $\exp(-\emptyset(\varepsilon))$ method, conformable derivative, exact solution.

REFERENCES

[1] R. Khalil, M. Al Horani, A. Yousef, M. Sababheh, A new definition of fractional derivative, Journal of Computational and Applied Mathematics, 264 (2014), 65-70.

[2] T. Abdeljawad, On conformable fractional calculus. Journal of Computational and Applied Mathematics, 279 (2015), 57-66.

[3] A. Atangana, D. Baleanu, A. Alsaedi, New properties of conformable derivative, Open Mathematics, 13 (2015), 2008.

[4] A. Akbulut, M. Kaplan, F. Tascan, The investigation of exact solutions of nonlinear partial differential equations by using $\exp(-\emptyset(\varepsilon))$ method, Optik 132 (2017), 382-387.

Dynamic Mode Decomposition of Wake Flow

Mustafa Hicret YAMAN [1], **Gamze YÜKSEL**[2]
1 Department of Mathematics, Faculty of Science, Mugla Sitki Kocman University,
hicretyaman@gmail.com
2 Department of Mathematics, Faculty of Science, Mugla Sitki Kocman University,
ngamze@mu.edu.tr

ABSTRACT

Dynamic Mode Decomposition (DMD) is a powerful technique in Fluid Mechanics that can extract fundamental information from snapshots of the fluid flows. Dynamic Mode Decomposition algorithm reveals the spatio-temporal coherent structures of the model by using only the data, without the governing equations so DMD is a full data-driven technique [1].

DMD has a variety of uses and interpretations. Basically, the DMD algorithm generally appears to have three main uses. These are; Diagnostics, State estimation and future-state prediction and Control [2,3]. In the current study diagnostics purpose is used. So that, the aim of this study is to diagnose the flow characteristic of the model and reconstruct the flow by revealing the pattern found in the flow by using DMD. DMD is applied to a fluid flow model called “Cylinder Wake Flow” obtained by the flow past a circular cylinder and turbulent wake flow field is investigated.

In this investigation, firstly, desired dataset and snapshots of the flow are obtained from the computational fluid dynamics (CFD) simulation which is performed on open source software; OpenFoam. Secodly, DMD algorithm is used to model reduction and generate the reconstructed flow. Moreover, the dominant modes of the flow, namely the flow characteristics, are examined. Finally, some conclusions is driven by comparing the reconstructed flow and the original flow.

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Key Words: Dynamic Mode Decomposition, Cylinder Wake Flow, Reduced Order Model.

REFERENCES

[1] J. Nathan Kutz, et al., Dynamic Mode Decomposition: Data-Driven Modeling of Complex Systems. SIAM 2016.

[2] P. J. Schmid. Dynamic mode decomposition of numerical and experimental data. Journal of Fluid Mechanics, 656:5–28., 2010

[3] Jonathan H. Tu, Clarence W. Rowley, Dirk M. Luchtenburg, Steven L. Brunton, J. Nathan Kutz, On Dynamic Mode Decomposition: Theory and Applications, 2014.

Some problems with stabilization of linear systems

Şerife YILMAZ[1]

1. Faculty of Education, Burdur Mehmet Akif Ersoy University, Istiklal Campus 15030 Burdur, Turkey serifeyilmaz@mehmetakif.edu.tr

ABSTRACT

A discrete-time linear system whose characteristic polynomial has all roots in the open unit disk is called Schur stable system. In this study, determination problems of Schur stability of linear systems are addressed by considering the geometric and topological properties of Schur stable monic polynomials space. The main handicap of the stability and stabilization problems is the non-convexity of the stability domain. The set of n-th order Schur stable monic polynomials is contained by the convex hull of $(n+1)$ known monic polynomials. On the other hand, a multilinear transformation can be defined from the n-dimensional open cube $(-1,1)^n$ onto this space of Schur stable monic polynomials. Using such a map, we construct polytopes that contained by the stability set. Given a linear system, we propose an algorithm that determines the stabilizing parameters if exists. For this purpose, an inner polytope is established with a chosen stable polynomial and it is determined whether this polytope intersects with the affine set corresponds to the linear system. The problem of determining the parameters that stabilizing two different linear systems at the same time is also important. Simultaneously stabilization problems can be also solved by our results. By using the multilinear transformation described here, inner polytopes can be defined in the space of non-monic polynomials and the stabilization problem can be evaluated. A number of examples are provided.

Key Words: Stabilization, discrete linear systems, Schur stability.

REFERENCES

[1] A.T. Fam and J.S. Meditch, A Canonical Parameter Space for Linear Systems Design, IEEE Transactions on Automatic Control, 23(3) (1978), 454-458.

[2] B.R. Barmish, New Tools for Robustness of Linear Systems, Macmillan, New York, 1994.

[3] Ü. Nürges, New Stability Conditions via Reflection Coefficients of Polynomials, IEEE Transactions on Automatic Control, 50(9) (2005), 1354-1360.

[4] Ş. Yılmaz, Polytopes for Discrete Systems by Using Box Coefficients, Circuits, Systems, and Signal Processing, 41 (2022), 789-804.

A New Formula for the Operational Matrix of the Fractional Derivative of the Chebyshev Polynomials

Dilek Varol[1]
1. Department of Mathematics, Faculty of Science, Pamukkale University, Denizli, TURKEY
dvarol@pau.edu.tr

ABSTRACT

In this study, a new formula for the operational matrix of the fractional derivative of the Chebyshev polynomials has been introduced. Here, the fractional derivative has been considered in the Caputo sense. The operational matrix has given the relation between the Chebyshev polynomials and the Caputo fractional derivative of them using the formula of the fractional derivative of the Chebyshev polynomials in terms of Chebyshev polynomials. In addition, this matrix has been compared with the other operational matrices of Chebyshev polynomials.

Key Words: Caputo fractional derivative, Chebyshev polynomials, operational matrix.

REFERENCES

[1] I. Podlubny, Fractional Differential Equations, Academic Press, USA, 1999.

[2] J.C. Mason and D.C. Handscomb, Chebyshev polynomials, Chapman and Hall/CRC, 2002.

Inverse Spectral and Inverse Nodal Problems For Singular Diffusion Equation

Rauf AMİROV [1] and Sevim DURAK[2]

1 Prof.Dr, Sivas Cumhuriyet University, Faculty of Science, Department of Mathematics
emirov@cumhuriyet.edu.tr

2 Asst.Prof..Dr, Sivas Cumhuriyet University, Faculty of Science, Department of Mathematics
sdurak@cumhuriyet.edu.tr

ABSTRACT

The [1] study contains detailed information about solvable models of quantum mechanics. As can be seen, these models are generally expressed with Hamilton operators or Schrödinger operators with singular coefficients. Many of the problems expressed by these models are related to the solution of spectral inverse problems for differential operators with singular coefficients. However, many problems of mathematical physics are reduced to the study of differential operators whose coefficients are generalized functions.

For example, the stationary vibrations of a spring-tied homogeneous wire fixed at both ends, density $R'(x) = a\delta(x - x_0)$ ($\delta(x)$-Dirac function) and stiffness $R(x)$ at the x_0 point, whose domain set is

$$D(L) = \left\{ \begin{array}{r} y(x) \in W_2^2[0,1]: y'(x_0 + 0) - y'(x_0 - 0) = ay(x_0), x_0 \in (0,1); \\ y(0) = 0 = y(1) \end{array} \right\}$$

and is expressed by the differential operator given as $L = (d^2/dx^2)$ in $L_2[0,1]$ Hilbert space. There is detailed information about the correct (regular) definition of such operators and the examination of their spectral properties in [2] studies.

We consider the following quadratic pencils of Sturm-Liouville equation of the form

$$\ell y \coloneqq -y'' + [\beta\lambda p(x) + q(x)]y = \lambda^2 y, \quad x\epsilon[0,\pi] \setminus \{a\}$$

with the boundary conditions

$U(y) := y'(0) - hy(0) = 0$

$V(y) := y'(\pi) - hy(\pi) = 0$,where q(x) is a real function belonging to the space $L_2[0,\pi]$,λ is a spectral parameter, $p(x)\epsilon W_2^{-1}[0,\pi]$, h, a, H and β are real numbers.

In the presented study, firstly, the characteristics of singular diffusion operators are given. Then, the properties of the solutions of the diffusion differential equation with singular coefficients are examined. The behavior of the eigenvalues and eigenfunctions of the given diffusion operator is examined by using the properties of the solutions. With the help of eigenvalues and normalizing numbers, some properties of the characteristic function and the uniqueness theorems for the solution of some spectral inverse problems are proved by using these properties. Finally, the behavior of the nodal points is examined for the given problem. And with their help, an algorithm is given for solving the inverse nodal problem and determining the solution. This type of problems, namely, half-axis inverse spectral problems for diffusion operators with singular coefficients and inverse spectral problems according to two spectral are investigated in [3], [4] studies, respectively.

Key Words: Singular diffusion equation, nodal points, inverse nodal problem.

REFERENCES

[1] S. Albeverio, F. Gesztesy, R. Hoegh-Krohn and H. Holden, Solvable models in quantum mechanics, springer, New York-Berlin, xiv+452 pp., (1988).

[2] A. M. Savchuk, On the Eigenvalues and Eingenfunctions of the Sturm-Liouville Operator with Singular Potential, Mathematical Notes, Vol. 69, No.2, (2001).

[3] N. Pronska, Reconstruction of energy-dependent Sturm-Liouville equations from two spectra, arXiv: 1205,4499, v1 (Math. SP), (2012).

[4] I. M. Guseinov and L. I. Mammadova, Reconstruction of the Diffusion Equation with Singular Coefficients for Two Spectra. Doklady Mathematics, Vol.90. No.1, pp.401-404, (2014).

Inverse Nodal Problems For Sturm-Liouville Operators With Singular Coefficient

Sevim DURAK[1] and Esra ÇONGAR [2]

1 Asst.Prof..Dr, Sivas Cumhuriyet University, Faculty of Science, Department of Mathematics
sdurak@cumhuriyet.edu.tr

2 Sivas Cumhuriyet University, Graduate School of Natural and Applied Sciences, Mathematics
avcesra@hotmail.com.

ABSTRACT

Schrödinger equation with singular potential, which is one of the basic equations of quantum theory, represents an important class of differential equations that have applications in many fields of engineering sciences. These types of singular differential operators, which we encounter in wave theory, heat transfer and determination of Coulomb type potential fields, have an important place in solving wave amplitudes, wavelengths and behavior of waves in different potential fields and many similar problems. Therefore, learning inverse nodal problems for singular Schrödinger operators is important in terms of applied sciences.

The inverse problems of spectral analysis are concerned with constructing the operator using spectral data. Here by spectral data is meant the eigenvalue, normalizing numbers and spectral function. In some studies of inverse problems, uniqueness theorems are given which show that the spectral characteristics determine the operator in a unique way. Initially, inverse nodal problems were discussed by Hald and McLaughlin $[1]$ in their study published in 1989. In this study, inverse nodal problems for regular and weakly singular Sturm-Liouville operators are discussed, uniqueness theorems for their solution and an algorithm for determining the potential are proposed. After this study, many studies have been carried out on the inverse nodal problems for the Sturm-Liouville and Dirac operators ($[2-4]$).

In our study, the boundary value problem given with the differential equation

$$\ell y := -y'' + q(x)y = \lambda^2 \delta(x) y$$

and

$$y'(0) - hy(0) = 0$$
$$y'(\pi) + Hy(\pi) = 0$$

boundary conditions, where , $q(x)\epsilon L_2[0,\pi]$ is a bounded and measurable function, is discussed. Here, λ is a spectral parameter, $\delta(x)=\begin{cases}1, x<a\\ \beta^2, x>a\end{cases}$, with $\beta\epsilon\mathbb{R}_+\setminus\{1\}$.

In the present study, properties of solutions of singular Sturm-Liouville differential equations, behavior of eigenfunctions, asymptotic behavior of eigenvalues and nodal points, theorems related to the solution of inverse nodal problem are given. In addition, an algorithm is proposed for the solution of the inverse nodal problem. Finally, by examining the behavior of the nodal points, the following theorem is proved.

Theorem: Let's take the sequences $\left\{x_{jn(1)}^{j}\right\}_{n\geq 1}\subset D_2$ and $\left\{h_{jn(1)}^{j}\right\}_{n\geq 1}$ as $\lim_{n\to\infty} x_{jn(1)}^{j}=x$ and $\lim_{n\to\infty} h_{jn(1)}^{j}=h_0$. Then

$$f(x):=\lim_{n\to\infty}\left(x_{jn(1)}^{j}-\left(a-\frac{a}{\beta}\right)\right)$$

$$g(x):=\lim_{n\to\infty}\left[\left(x_{jn(1)}^{j}-2a\beta^{-}-\frac{h_n\left(x_{jn(1)}^{j}-2a\beta^{-}\right)}{(\beta^{+})^2 n\pi}-\frac{j-\frac{1}{2}}{n}-\frac{\beta^{+}}{\beta^{+}n\pi}\right)(n\pi)^2\right]$$

limits are exists and $f(x)=\frac{x}{\beta}\gamma(\pi)$, $g(x)=\frac{\beta^{-}h_0}{\beta(\beta^{+})^3}+Q(x)+h_0D(x)$ equalities are true.

Keywords: Sturm-Liouville operators, nodal points, inverse problem.

REFERENCES

[1] O.H. Hald, J.R. McLaughlin, Solutions of inverse nodal problems, Inverse Problems, 5, 539-577, (1989).

[2] C.T Shieh, V.A. Yurko, Inverse nodal and inverse spectral problems for discontinuos boundary value problems, J. Math. Anal. Appl. 347,266-272, (2008).

[3] C.K. Law, C.F. Yang, Reconstructing the potential function and its derivatives using nodal data, Inverse Problems, 14(2):299-312, (1998).

[4] J.R. McLaughlin, Inverse spectral theory using nodal points as data-a uniqueness result, Journal of Differential Equations, 73, 354-262, (1988).

Some Properties of Matrix Family Which Consist of Convex Combinations

<u>Güner Topcu</u> [1] and Kemal Aydın [2]
1 Selçuk University, guner.ozturk@selcuk.edu.tr
2 Selçuk University, kaydin@selcuk.edu.tr

ABSTRACT

In this study, some Schur stable matrix family $\mathcal{C}$ is constructed from convex combinations of matrices $A \in S_N = \{A \in M_N(\mathbb{C}) | |\lambda_i(A)| < 1\}$ and $B \in M_N(\mathbb{C})$. In the literature, a necessary and sufficient condition for matrix A to be Schur stable is that the Lyapunov matrix equation $A^*HA - H + I = 0$ must have a matrix $H = H^* > 0$, according to Lyapunov's theorem [4]. Moreover, the parameter ω which indicates the quality of Schur stability of the matrices, is defined as $\omega(A) = \|H\| \geq 1$. If the condition $\omega(A) < \infty$ applies then A is Schur stable, on the other hand it is written as "$\omega(A) = \infty$" if A is not Schur stable [1,3,4]. $\omega^*(> 1)$ parameter specified by users, if $\omega(A) \leq \omega^*$ then the matrix A is ω^*-Schur stable matrix. In this study, matrix family $\mathcal{C}$ was introduced with convex combinations of the matrices $A \in S_N$ and $B \in M_N(\mathbb{C})$. Some properties about this family and their proofs were introduced. The continuity theorems were also given [2,5]. Some theorems and results about the Schur stability (ω^*-Schur stability) of this family were given. With given theorems, the interval $\mathcal{J}$ ($\mathcal{J}^*$) was obtained. This interval guarantees the Schur stability (ω^*-Schur stability) of the matrix family $\mathcal{C}$. After all, the illustrative examples related to this family were given.

Key Words: Schur stability, continuity theorems, convex combination

REFERENCES

[1] K. Aydın, H. Bulgak, G. V. Demidenko, Numeric characteristics for asymptotic stability of solutions to linear difference equations with periodic coefficients, *Siberian Mathematical Journal*, 2000, 41.6: 1227-1237 (2000); translation in Sib. Math. J. 41.

[2] K. Aydin, H. Bulgak, G. V. Demidenko, Asymptotic stability of solutions to perturbed linear difference equations with periodic coefficients, Siberian Mathematical Journal, 43(3) (2002), pp. 389-401

[3] A. Ya. Bulgakov, S. K. Godunov, Circle dichotomy of the matrix spectrum, Siberia Math. J., 29, No: 5 (Novosibirsk 1988), pp. 59-70.

[4] H. Bulgak, Pseudoeigenvalues, spectral portrait of a matrix and their connections with different criteria of stability, Error Control and Adaptivity in Scientific Computing, (1999), pp. 95-124

[5] A. Duman, K. Aydın, Sensitivity of Schur stability of systems of linear difference equations with constant coefficients, Scientific Research and Essays, 6(28) (2011), pp. 5846- 5854

Some Properties of Matrix Family Which Consist of Linear Combinations

Güner Topcu [1] **and Kemal Aydın** [2]
1 Selçuk University, guner.ozturk@selcuk.edu.tr
2 Selçuk University, kaydin@selcuk.edu.tr

ABSTRACT

In this study, some Schur stable matrix family $\mathcal{L}$ is constructed from linear combinations of matrices $A \in S_N = \{A \in M_N(\mathbb{C}) | |\lambda_i(A)| < 1\}$ and $B \in M_N(\mathbb{C})$. In the literature, a necessary and sufficient condition for matrix A to be Schur stable is that the Lyapunov matrix equation $A^*HA - H + I = 0$ must have a matrix $H = H^* > 0$, according to Lyapunov's theorem [4]. Moreover, the parameter ω which indicates the quality of Schur stability of the matrices, is defined as $\omega(A) = \|H\| \geq 1$. If the condition $\omega(A) < \infty$ applies then A is Schur stable, on the other hand it is written as "$\omega(A) = \infty$" if A is not Schur stable [1,3,4]. $\omega^*(> 1)$ parameter specified by users, if $\omega(A) \leq \omega^*$ then the matrix A is ω^*-Schur stable matrix. In this study, matrix family $\mathcal{L}$ was introduced with linear combinations of the matrices $A \in S_N$ and $B \in M_N(\mathbb{C})$, respectively. Some properties about this family and their proofs were introduced. The continuity theorems were also given [2,5]. Some theorems and results about the Schur stability (ω^*-Schur stability) of this family were given. With given theorems, the interval $\mathcal{J}$ ($\mathcal{J}^*$) was obtained. This interval guarantees the Schur stability (ω^*-Schur stability) of the matrix family $\mathcal{L}$. After all, the illustrative examples related to the this family were given.

Key Words: Schur stability, continuity theorems, linear combination

REFERENCES

[1] K. Aydın, H. Bulgak, G. V. Demidenko, Numeric characteristics for asymptotic stability of solutions to linear difference equations with periodic coefficients, *Siberian Mathematical Journal*, 2000, 41.6: 1227-1237 (2000); translation in Sib. Math. J. 41.

[2] K. Aydin, H. Bulgak, G. V. Demidenko, Asymptotic stability of solutions to perturbed linear difference equations with periodic coefficients, Siberian Mathematical Journal, 43(3) (2002), pp. 389-401

[3] A. Ya. Bulgakov, S. K. Godunov, Circle dichotomy of the matrix spectrum, Siberia Math. J., 29, No: 5 (Novosibirsk 1988), pp. 59-70.

[4] H. Bulgak, Pseudoeigenvalues, spectral portrait of a matrix and their connections with different criteria of stability, Error Control and Adaptivity in Scientific Computing, (1999), pp. 95-124

[5] A. Duman, K. Aydın, Sensitivity of Schur stability of systems of linear difference equations with constant coefficients, Scientific Research and Essays, 6(28) (2011), pp. 5846- 5854

On the Existence and Uniqueness of an Initial-Boundary Problem for a Semilinear Fractional Diffusion Equation

Özge Arıbaş [1], İsmet Gölgeleyen [1] and Mustafa Yıldız [1]
1 Department of Mathematics, Faculty of Science, Zonguldak Bülent Ecevit University, Incivez, Zonguldak, 67100, Turkey. ozge.aribas@beun.edu.tr, ismet.golgeleyen@beun.edu.tr and mustafa.yildiz@beun.edu.tr.

ABSTRACT

In this work, we consider an initial-boundary value problem for a semilinear fractional diffusion equation on a bounded domain. The solution of the problem depends on both time and space. The fractional derivative in the equation is Caputo type. Fractional diffusion equations have many applications in science and technology.

Our aim is to investigate the existence and uniqueness of solution of the problem. The main tool which we use for this purpose is the Fourier method. We solve the problem with using superposition of the solutions for the boundary value problem. Then, we determine the functions dependant on the time variable, which also satisfy the initial condition. This is a fundamental technique for finding solutions of initial-boundary value problems for partial differential equations. In the works of [2], [3] and [4], this method was applied and generalized to various initial-boundary value problems that involve a fractional order diffusion equation. At the end of the evaluation, the obtained solution of the problem is a weak solution. In our study, we use this approach for a more general problem. For the proof of our main theorem, we use the method given by [2], which can be considered as a generalization of the proof of Picard's theorem for ordinary differential equations, [1]. We also employ a priori estimates in the space of square integrable functions in order to get our results.

Key Words: Semilinear fractional diffusion equation, initial-boundary value problem, existence, uniqueness.

REFERENCES

[1] H. Brezis. Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, New York, NY, 2011.

[2] B. Jin, Fractional Differential Equations An Approach via Fractional Derivatives, Springer Nature, Switzerland, 2021.

[3] Y. Luchko, Some uniqueness and existence results for the initial-boundary-value problems for the generalized time-fractional diffusion equation, Comput. Math. with Appl. 59 (2010),1766–1772.

[4] K. Sakamoto and M. Yamamoto, Initial value/boundary value problems for fractional diffusion-wave equations and applications to some inverse problems, J. Math. Anal. Appl. 382 (2011),426–447.

Spectral Expansion Formula for a Singular Sturm-Liouville Problem

Kh.R. Mamedov [1] and R.G. Farzullazadeh[2]
1 Igdir, Turkey, Faculty of Science and Literature, Department of Mathematics, Igdir University,
hanlar.residoglu@igdir.edu.az
2 Lankaran, Azerbaijan, Faculty of Natural Sciences, Department of Mathematics and Informatics, Lankaran State University,
framin1992@mail.ru

ABSTRACT

We consider the boundary value problem generated by the equation

$$-\psi'' + q(x)\psi = \lambda^2\psi \,, \tag{1}$$

with the boundary condition

$$\left(b_0 + b_2\lambda^2\right)\psi'(0) - \left(a_0 + a_1\lambda + a_2\lambda^2\right)\psi(0) = 0 \,, \tag{2}$$

where λ is a complex parameter, b_j, a_j $(j = 0, 1, 2)$ are real numbers, $q(x)$ is a real valued function and $(1+x)q(x) \in L_1(0,\infty)$.

We study the spectral analysis of the boundary value problem (1), (2). Using the Jost function of the equation (1) and Titchmarsh's method [1], we construct the resolvent operator and obtain two-fold expansion formula according to the scattering data. The physical application of the expansion problem for the differential equation on the half line $[0,\infty)$ with the boundary condition on the spectral parameter is considered in T.Regge's works [2, 3]. Expansion formulas according to the eigenfunctions for the equation (1) with different types of boundary conditions are obtained in [4, 5] and other studies. For the reason, scattering data of the problem (1), (2) is differently obtained.

Key Words: Expansion formula, eigenfunction, scattering data.

REFERENCES

[1] E.C. Titchmarsh, Eigenfunction expansions associated with second-order differential equations, Oxford Clarendon Press, 1958.

[2] C. T. Fulton, Singular eigenvalue problems with eigenvalue parameter contained in the boundary conditions, Proceedings of the Royal Society of Edinburgh, (1980), 87, p. 1-34.

[3] T. Regge Analytical properties of the scattering matrix, Nuovo Cimento, (1958), 8, p. 671-679.

[4] Kh. R. Mamedov, Uniqueness of the solution to the inverse problem of scattering theory for the Sturm-Liouville operator with a spectral parameter in the boundary condition, Mathematical Notes, (2003), 74, p. 136-140.

[5] V. Ala and Kh.R. Mamedov, Expansion formula for a singular Sturm-Liouville operator, Uzbek Mathematical, (2020), 1, p. 10-20.

Comparison Of Semi-Analytical Methods With Solving Surge Tank Problem

Kürşat Canevi [1], Sema Servi [2] and Galip Oturanç[3]
1 Karamanoğlu Mehmetbey University, Faculty of Science, Department of Mathematics, Karaman/TURKEY, kursatcanevi@gmail.com
2 Selcuk University, Faculty of Technology, Department of Computer Engineering, Konya/TURKEY, semaservi@selcuk.edu.tr
3 Karamanoğlu Mehmetbey University, Faculty of Science, Department of Mathematics, Karaman/TURKEY, goturanc@kmu.edu.tr

ABSTRACT

Hydroelectric power plants are substantial alternative energy source among renewable energy sources. While these power plants are working, sudden pressure changes occur in the water transfer system that reveal a concept called water hammer. In these power plants, a structure called surge tank is usually built to prevent the water hammer from damaging any element in the water transfer system. The surge tank needs to be constructed according to the physical characteristics of the dam. The engineering problem revealed in the construction of the surge tank is expressed by a nonlinear differential equation system. This equation system does not have analytical solution. In this paper, four different numerical methods were used to find the solution of the surge tank problem, which were expressed as an initial value problem. The methods those are used and to be compared are differential transform method, Adomian decomposition method, variational iteration method and Runge-Kutta-Fehlberg method. Calculation algorithms were executed with Maple software. It has been observed that the obtained solutions with all four methods are consistent.

Key Words: Differential transform method, Adomian decomposition method, variational iteration method, Runge-Kutta-Fehlberg method, surge tank.

REFERENCES

[1] S. Crandall, Engineering Analysis: A Survey of Numerical Procedures. New York: McGraw-Hill, 1956.

[2] G. Adomian, A new approach to nonlinear partial differential equations, Journal of Mathematical Analysis and Applications (1984), 420-434.

[3] J. He, A new approach to nonlinear partial differential equations. Communications in Nonlinear Science and Numerical Simulation (1997), 203-205.

[4] S. Wang and J. He, Variational iteration method for solving integro-differential equations. Physics Letters A (2007), 188-191.

[5] J. He, Variational iteration method – a kind of non-linear analytical technique: some examples. International Journal of Non-Linear Mechanics (1999), 699-708.

[6] J. K. Zhou, Differential transformation and its applications for electrical circuits. Wuhan: Huazong University Press, 1986.

[7] E. Fehlberg, Klassische Runge-Kutta-Formeln vierter und niedrigerer Ordnung mit Schrittweiten-Kontrolle und ihre Anwendung auf Wärmeleitungsprobleme. Computing 6 (1970), 61-71.

[8] Y. Keskin and G. Oturanç, Diferansiyel Dönüşüm Yöntemiyle Diferansiyel Denklemlerin Çözülmesi. Konya: Selçuk Üniversitesi Fen Fakültesi Matematik Bölümü, 2010.

[9] A. Kurnaz and G. Oturanç, The differential transform approximation for the system of ordinary differential equations. International Journal of Computer Mathematics (2005), 709-719.

[10] C. Chen and S. Ho, Application of Differential transformation to eigenvalue problems. Applied Mathematics and Computation (1996), 173-188.

[11] C. Chen and S. Ho, Solving partial differential equations by two-dimensional differential transform method. Applied Mathematics and Computation (1999), 171-179.

[12] M. Abdou and A. Soliman, New applications of variational iteration method. Physica D: Nonlinear Phenomena (2005), 1-8.

[13] A. Arıkoğlu and İ. Özkol, Solution of boundary value problems for integro-differential equations by using differential transform method. Applied Mathematics and Computation (2005), 1145-1158.

[14] J. Hunsaker and B. Rightmire, Engineering Applications of Fluid Mechanics. New York: McGraw-Hill Book Company Inc., 1947.

[15] G. Rich, Hydraulic Transients. New York: McGraw-Hill Book Company Inc., 1951.

[16] M. Inokuti, General Use of the Lagrange Multiplier in Nonlinear Mathematical Physics. N. Nasser içinde, Variational Method in the Mechanics of Solid (s. 156-162), (1978), Oxford: Pergamon Press.

[17] K. Canevi, (2022). Yarı Analitik Yöntemlerin Hidroelektrik Santrali Denge Bacası Probleminin Çözümü ile Kıyaslanması. Karamanoğlu Mehmetbey University, Turkey.

On the Scattering Problem for a Discontinuons Boundary Value Problem

Kh.R. Mamedov [1] and Sh.R. Alizadeh[2]
1 Igdir, Turkey, Faculty of Science and Literature, Department of Mathematics, Igdir University,
hanlar.residoglu@igdir.edu.az
2 Lankaran, Azerbaijan, Faculty of Natural Sciences, Department of Mathematics and Informatics, Lankaran State University,
shabnuralizade@gmail.com

ABSTRACT

We consider the singular boundary value problem on the half line $[0,\infty)$

$$-u'' + q(x)u = zu, \tag{1}$$

$$u'(0) + (b_1 + b_2 z)u(0) = 0 \tag{2}$$

and

$$\begin{aligned} u(a-0) &= \alpha\ u(a+0), \\ u'(a-0) &= \alpha^{-1}u(a+0), \end{aligned} \tag{3}$$

Where $z = \lambda^2$ is a spectral parameter, $q(x)$ is a real valued Lebesgue measurable function satisfying

$$\int_0^\infty (1+x)|q(x)|dx < \infty\ , \tag{4}$$

It is well known, in the classical case $(b_2 = 0)$ the direct and inverse problem of scattering theory for the boundary value problem (1), (2) was completely solved in [1], [2]. Similar problem for the equation (1) when the boundary conditions dependent on the spectral parameter are examined in [3,4,5] and spectral analysis on the half line was studied in [6].

The main aim of this paper is to show the Marchenko's method applying to the boundary value problem (1), (2) with discontinuity conditions (3) at the interior point $x = a \in (0,\infty)$.

Key Words: Spectral parameter, boundary value problem, scattering theory.

REFERENCES

[1] V.A. Marchenko, Sturm-Liouville operators and applications, Birkhauser Verlag, 1986.

[2] B.M. Levitan, Inverse Sturm-Liouville problems, Utrect, 1987.

[3] Kh. R. Mamedov, Uniqueness of the solution to the inverse problem of scattering theory for the Sturm-Liouville operator with a spectral parameter in the boundary condition, Mathematical Notes, (2003), 74, p. 136-140.

[4] Kh.R. Mamedov, On the inverse problem for Strum-Liouville operator with a nonlinear spectral parameter in the boundary condition, Journal of the Korean Mathematical Society, (2009), 46, p. 1243-1254.

[5] Y.Yang. and G. Wei, Inverse scattering problems for Sturm- Liouville operators with spectral parameter dependent on boundary conditions, Mathematical Notes, (2018), 103 (1), p. 59-66.

[6] C. T. Fulton, Singular eigenvalue problems with eigenvalue parameter contained in the boundary conditions, Proceedings of the Royal Society of Edinburgh, (1980), 87, p. 1-34.

Estimates of singular numbers (s-numbers) and eigenvalues of a mixed elliptic-hyperbolic type operator with parabolic degeneration

Muratbekov M.B., Muratbekov M.M.
Department of Higher Mathematics and Mathematics Teaching Methodology, Taraz State Pedagogical University, Taraz, Kazakhstan

ABSTRACT

Consider the mixed type differential operator
$Lu = k(y)u_{xx} - u_{yy} + b(y)u_x + q(y)u$,

which is initially defined with $C_{0,\pi}^{\infty}(\overline{\Omega})$, where $\overline{\Omega} = \{(x, y): -\pi \le x \le \pi,\ -\infty < y < \infty\}$, $C_{0,\pi}^{\infty}$ is a set of infinitely differentiable functions with compact support of a variable and satisfy the conditions:

$u_x^{(i)}(-\pi, y) = u_x^{(i)}(\pi, y) \qquad i = 0, 1$.

Regarding the coefficient $k(y)$, with supposition that $k(y)$ satisfies the condition:

a) $|k(y)| \ge 0$ is a piecewise continuous and bounded function in $R = (-\infty, \infty)$.

The coefficients $b(y)$ and $q(y)$ are continuous functions in R and can be unbounded at infinity.

The L operator is susceptible to closure in space $L_2(\Omega)$ and the closure is also denoted by L.

Taking into consideration certain constraints on the coefficients $b(y)$ и $q(y)$, apart from the above-mentioned conditions, the existence of a bounded inverse operator is proved in this paper; a condition guaranteeing compactness of the resolvent kernel is detected; and we also obtained two-sided estimates for singular numbers (*s*-numbers). Here we note that the estimate of singular numbers (*s*-numbers) shows the rate of approximation of the resolvent of the operator *L* by linear finite-dimensional operators. It is given an example of how the obtained estimates for the *s*-numbers enable to identify the estimates for the eigenvalues of the operator L. We note that the above results are apparently obtained for the first time for a mixed-type operator in the case of an unbounded domain with rapidly oscillating and greatly growing coefficients at infinity.

References

[1] Muratbekov M.B., Muratbekov M.M. On the compactness of the resolvent of a Schrodinger type singular operator with a negative parameter // Chaos, Solitons and Fractals, Vol. 151 (2021), 111248.

[2] Muratbekov M.B., Muratbekov M.M. One-dimensional Schrodinger operator with a negative parametres and its applications to the study of the approximation numbers of singular hyperbolic operators // Filomat, 32:3 (2018), pp. 785-790.

GEOMETRY

On Dimensions of Invariant Tensor Fields on MWH Spaces with Subgroup of Given Type

R.M. Surmanidze[1]

1. Faculty of Exact and Natural Sciences, Ivane Javakhishvili Tbilisi State University, 13 University St., Tbilisi 0177, Georgia.
Email: ruslan.surmanidze@tsu.ge

ABSTRACT

The object of our research is a Homogeneous Riemann spaces, which held an important place in geometry. To study invariant properties of a geometric object was always important in mathematics. The theory of homogeneous spaces also closely connected with the theory of invariant tensors. For example, homogeneous symmetric spaces and invariant tensors with respect to representations of semi-simple Lie groups.

An isotropically irreducible Manturov–Wolf homogeneous space is a homogeneous Riemannian space whose isotropy group is irreducible [1, 2]. For these spaces in the tangent space only trivial subspaces are invariants of the isotropy group. Invariants with respect to the group of motion of isotropically irreducible homogeneous spaces, can be described in terms of invariant tensor fields on these spaces. Our construction, allows us to construct tensors invariant with respect to the isotropy group.

We considere invariant tensors fields on the Manturov–Wolf homogeneous space in case when isotropic representation has the type $SL(2) \otimes SO(2n)$, $n > 3$ [3].

Received the formula of expansion of the tensor square of an isotropic representation of the given MWH space onto direct sum of irreducible representations. Proven the main theorem on the dimensions of spaces of invariant tensor fields valency 2, 3 and 4. The received results has as being, also, of applied character.

Key Words: Homogeneous Riemannian space, invariant tensor field, Isotropy group.

REFERENCES

[1] O. V. Manturov, Homogeneous Riemannian spaces with an irreducible rotation group. Trudy Sem. Vektor. Tenzor. Anal. (Russian) 13 (1966), 68-145.

[2] J. A. Wolf, The goemetry and structure of isotropy irreducible homogeneous spaces. Acta Math. 120 (1968), 59-148.

[3] R. Surmanidze, On Dimensions of Invariant Tensor Fields on MWH Spaces with Subgroup of Type SL(2) $\otimes$ SO(2n), n > 3. Journal of Computational Mathematica, Vol. 6, No 1 (2022): Spacial Issue in Honor of Professor Ravi P. Agarwal, 222-232.

On addition formulas related to elliptic integrals

Malkhaz Bakuradze[1]

1. Tbilisi State University, faculty of exact and natural sciences, 0167, Tbilisi, Georgia
email: malkhaz.bakuradze@tsu.ge

ABSTRACT

Addition theorems offer a means of determining the value of the function for the sum of two quantities as arguments, when the values of the function for each argument is known. The simplest examples are the well-known addition formulas for the functions e^u and $\sin u$.

The elliptic sine $f(u) = sn(u)$, satisfies Euler's addition theorem

$$f(u+v) = \frac{f(u)R(f(v)) + f(v)R(f(u))}{1-k^2 f(u)^2 f(v)^2}, \quad \text{where} \quad R(u) = \sqrt{(1-u^2)(1-k^2u^2)}$$

and k is some parameter called modulus.

We review some other interesting examples, such as Baker-Akhiezer functions and the exponent series of the Ochanine elliptic genus.

As a main result we provide certain addition formulas for the elliptic integrals corresponding to the general elliptic genus with logarithm having differential $\frac{dt}{dR(t)}$, where $R(t)$ is a monic polynomial of degree 4. Our result specializes in Euler's addition theorems for elliptic integrals of the first and second kind and in the examples above.

The proofs are given in terms of the formal group laws.

Key Words: Formal group law, elliptic integral.

REFERENCES

[1] M. Bakuradze, V. V. Vershinin, On addition theorems related to elliptic integrals, Proceedings of the Steklov Institute of Mathematics 305 (1), (2019), 22-32

A Survey on Non-null Slant Ruled Surfaces

Emel Karaca[1]

1. Department of Mathematics, Ankara Hacı Bayram Veli University, Ankara, Turkey
emel.karaca@hbv.edu.tr

ABSTRACT

In this study, exploiting the definition of non-null slant ruled surface, which is a ruled surface whose Frenet vectors make a constant angle with some fixed directions, in literature, the non-null slant ruled surface acquired by the striction curve of the natural lift curve has been defined with the new perspective in Minkowski 3-space E_1^3. In order to construct this definition, the correspondence between the subsets of the tangent bundles for pseudo-spheres and E. Study mapping has been considered. This mapping says that there exists an isomorphism between the curves on the tangent bundles of peudo-spheres and the spacelike or timelike slant ruled surfaces. This mapping has been modified fort he non-null slant ruled surfaces generated by the striction curve of the natural lift curve. $\overline{q}, \overline{h}$ and $\overline{a}$ spacelike or timelike slant ruled surfaces have been introduced in E_1^3. Furthermore, some characterizations for $\overline{q}, \overline{h}$ and $\overline{a}$ spacelike or timelike slant ruled surfaces generated by the striction curve of the natural lift curve have been denoted. Several significant theorems and remarks have been mentioned in detail. Then, some examples have been given to support the main results. Finally, these results have been discussed and open problems have also been mentioned.

Key Words: Non-null slant ruled surface, E. Study mapping, tangent bundle of pseudo-sphere.

REFERENCES

[1] R.A. Abdel-Baky, Slant ruled surface in Euclidean 3-space, Sci Magna. 9 (2013), 107-112.

[2] M. Bekar, F. Hathout and Y. Yaylı, Tangent bundle of pseudo-sphere and ruled surfaces in Minkowski 3-space, General Letters in Mathematics. 5 (2018), 58-70.

[3] E. Karaca and M. Çalışkan, Tangent bundle of unit 2-sphere and slant ruled surfaces, Filomat, (2020), (Submitted).

[4] E. Karaca and M. Çalışkan, Ruled surfaces and tangent bundle of pseudo-sphere of natural lift curves, Journal of Science and Arts. 20 (2020), 583-586.

[5] O. Kaya and M. Önder, Characterizations of slant ruled surfaces in the Euclidean 3-space, Caspian Journal of Mathematical Sciences. 6 (2017), 31-46.

[6] M. Önder, Non-null slant ruled surfaces, AIMS Mathematics. 4 (2019), 384-396.

On k-type spacelike slant helices due to Bishop frame in Minkowski space-time.

Hatice TOZAK [1], Cumali EKİCİ [2] and Yasin ÜNLÜTÜRK[3]
1 Strategy Department of Denizli Provincial Directorate of National Education, Denizli, TURKEY and hatice.tozak@gmail.com
2 Department of Mathematics-Computer, Eskişehir Osmangazi University, Eskişehir TURKEY cekici@ogu.edu.tr
3 Department of Mathematics, Faculty of Art and Science, Kırklareli University, Kırklareli, TURKEY yasinunluturk@klu.edu.tr

ABSTRACT

In this work, we study k-type (k=0,1,2,3) spacelike slant helices with non-zero Bishop curvature functions due to Bishop frame in Minkowski space-time. 0-type slant helices are general helices and 1-type slant helices are slant helices within the notation of this study. We prove that there are no 0-type spacelike slant helices due to Bishop frame in Minkowski space-time. Also, we characterize k-type spacelike slant helices and determine the axis of the curve in terms of Bishop curvatures in Minkowski space-time.

Key Words: General Helix, Slant Helix, Bishop Frame, Minkowski Space-time.

REFERENCES

[1] Ali and R. Lopez, Slant helices in Minkowski space E_1^3, J. Korean Math. Soc. 48 (2011), 159–167.

[2] B. O'Neill, Semi-Riemannian Geometry with Applications to Relativity, Academic press, New York, 1983.

[3] L. R, Bishop, There is more than one way to frame a curve, Amer. Math. Monthly, 82 (3) (1975), 246-251.

[4] M. Erdoğdu, Parallel frame of non lightlike curves in Minkowski space time, Int. J. Geom. Methods M., 12 (2015), 1550109-1-16.

[5] M. Turgut and S. Yilmaz, Some characterizations of type-3 slant helices in Minkowski space-time, Involve 2 (2009), 115–120.

[6] S. Izumiya and N. Takeuchi, New special curves and developable surfaces, Turkish J. Math. **28** (2004), 531–537.

[7] Y. Ünlütürk, S. Yılmaz, M. Çimdiker and S. Şimşek, Associated curves of non-lightlike curves due to the Bishop frame of type-1 in Minkowski 3-space, Advanced Modeling and Optimization, 20(1) (2018), 313-327.

[8] Y. Ünlütürk, H. Tozak and C. Ekici, On k-Type Slant Helices Due to Bishop Frame in Euclidean 4-Space E^4 , International J.Math. Combin. 1 (2020), 01-09.

Notes on Meta-Golden Riemannian Structures

Murat Altunbaş [1]
1 Department of Mathematics, Faculty of Art and Science, Erzincan Binali Yıldırım University, 24100, Erzincan, Turkey
Email: maltunbas@erzincan.edu.tr

ABSTRACT

In this work, we introduce a meta-golden Riemannian structure on manifolds and investigate the locally decomposability of the manifold with respect to this structure. We also give some examples for meta-golden structures on bundles.

Key Words: Chi ratio, meta-golden Riemannian manifold, pure tensor, bundle.

REFERENCES

[1] Gezer and Ç. Karaman, On Metallic Riemannian structures, Turkish J. Math. 39 (6) (2015), 954-962.

[2] C.E. Hretcanu and M. Crasmareanu, Metallic structures on Riemannian manifolds, Rev. de la Uni. Mat. Argentina 54 (2) (2013), 15-27.

[3] B. Şahin and F. Şahin, Meta golden Riemannian manifolds, Math. Meth. Appl. Sci. (2022), 1-11.

Translating Solitons on Null Direction in Minkowski 3 Space

Erdem Kocakuşaklı[1], **Miguel Ortega**[2]
1 Department of Mathematics, Sinop University, Sinop, Turkey ekocakusakli@sinop.edu.tr
2 Department of Geometry and Topology, University of Granada, Granada, Spain
miortega@ugr.es

ABSTRACT

In this talk, we deal with translating solitons of the mean curvature flow. In Minkowski 3-space; the definition of translating solitons with null direction is introduced. Firstly, we classify those which are graphical translation surfaces, obtaining spacelike and timelike, entire and not entire, complete and incomplete examples. Among them, all our timelike examples are incomplete. The second family consists of those which are invariant by a 1-dimensional subgroup of parabolic motions with null axis. The classification result implies that all examples of this second family have singularities.

Key Words: Translating solitons, Mean curvature flow, Null direction, Minkowski 3-Space.

REFERENCES

[1] E. Kocakusakli and M. Ortega, A zoo of translating solitons on a parallel light-like direction in Minkowski-3 space, Diff.geo. and its Appl., 78, 2021.

[2] J. H. S. de Lira, F. Martín, Translating solitons in Riemannian products, J. Differential Equations 266, no. 12, (2019), 7780-7812.

[3] R. López, M. Moruz, Translation and Homothetical surfaces in Euclidean Space with Constant Curvature, J. Korean Math. Soc. 52, No. 3, (2015), 523-535.

Mimkowski Difference of Cubes

Jalolxon Nuritdinov Tursunboy ogli [1]
1 National University of Uzbekistan, Tashkent, Uzbekistan
nuritdinovjt@gmail.com

ABSTRACT

The Minkowski difference of sets having the different configuration is basic to the treatment of a large class of problems often occurring in lots of interesting applications from a variety of areas, especially in problems of engineering design, in data classification, image analysis and processing, motion planning for robots, real-time collision detection, computer graphics, optimal control and many other front-line fields.

In [1], various geometric properties of the Minkowski difference on sets are studied in detail. In works [2],[3], the problems of finding the Minkowski difference of triangles and squares on the plane are covered. Below we present the results of our research on finding the Minkowski difference of cubes located in three-dimensional Euclidean space.

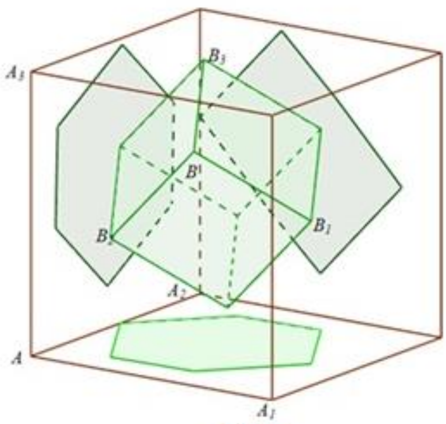

Fig.1

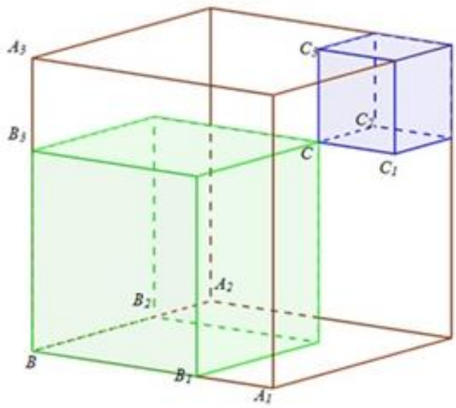

Fig.2

In order for the definition of a cube to be one-valued in Euclidean space $\mathbb{R}^3$, it is enough to give its four points that are not in the same plane and have common edges. Let the cubes C^A and C^B are given by vertices A, A_1, A_2, A_3 and B, B_1, B_2, B_3 respectively (Fig.1). We introduce the following denotations:

$$\overrightarrow{AA_1} = \vec{a}_1, \overrightarrow{AA_2} = \vec{a}_2, \overrightarrow{AA_3} = \vec{a}_3, \tag{1}$$

$$\overrightarrow{BB_1} = \vec{b}_1, \overrightarrow{BB_2} = \vec{b}_2, \overrightarrow{BB_3} = \vec{b}_3. \tag{2}$$

Using the defined operations on vectors, we can formulate all the diagonals of the cube C^B by vectors $\vec{b}_1, \vec{b}_2$ and $\vec{b}_3$,

$$\begin{aligned} \vec{d}_1 &= \vec{b}_1 + \vec{b}_2 + \vec{b}_3, \\ \vec{d}_2 &= -\vec{b}_1 - \vec{b}_2 + \vec{b}_3, \\ \vec{d}_3 &= -\vec{b}_1 + \vec{b}_2 + \vec{b}_3, \\ \vec{d}_4 &= \vec{b}_1 - \vec{b}_2 + \vec{b}_3. \end{aligned} \tag{3}$$

Theorem. In order for the difference $C^A \overset{*}{-} C^B$ not to be empty, it is necessary and sufficient that the length of the orthogonal projections of the vectors $\vec{d}_i, i = \overline{1,4}$ to the vectors $\vec{a}_1, \vec{a}_2$ and $\vec{a}_3$ should not be greater than the length of the vector $\vec{a}_1$.

Proof. There are two possible cases when calculating the difference $C^A \overset{*}{-} C^B$.

In the first case, all vectors $\vec{a}_1, \vec{a}_2$ and $\vec{a}_3$ are parallel to all vectors $\vec{b}_1, \vec{b}_2$ and $\vec{b}_3$, respectively (Fig.2),then the orthogonal projections of vectors $\vec{d}_i, i = \overline{1,4}$ onto vectors $\vec{a}_1, \vec{a}_2$ and $\vec{a}_3$ will be equal to the length of vector $\vec{b}_1$, that is

$$\left| proj_{\vec{a}_j} \vec{d}_i \right| = \left| \vec{b}_1 \right|, i = \overline{1,4}, j = \overline{1,3}. \tag{4}$$

It can be seen from the figure 2 that in this case, in order to place the cube C^B inside the cube C^A, that is, for the $C^A \overset{*}{-} C^B$ difference to exist, the relation $|\vec{a}_1| \geq |\vec{b}_1|$ is necessary and sufficient. This means that $\left| proj_{\vec{a}_j} \vec{d}_i \right| \leq |\vec{a}_1|, i = \overline{1,4}, j = \overline{1,3}$.

In the second case, at least one of the vectors $\vec{a}_1$, $\vec{a}_2$ and $\vec{a}_3$ is not parallel with one of the corresponding vectors $\vec{b}_1, \vec{b}_2$ and $\vec{b}_3$. In the general case, we assume that none of these vectors are parallel to each other (Fig.1). Then the lengths of the orthogonal projections of vectors $\vec{d}_i, i = \overline{1,4}$ to vectors $\vec{a}_1, \vec{a}_2$ and $\vec{a}_3$ are found using formula

$$\left| proj_{\vec{a}_j} \vec{d}_i \right| = \frac{\left| \left\langle \vec{a}_j, \vec{d}_i \right\rangle \right|}{\left| \vec{a}_j \right|}, i = \overline{1,4}, j = \overline{1,3}. \tag{5}$$

Here, $\left\langle \vec{a}_j, \vec{d}_i \right\rangle$ is a scalar product of vectors $\vec{d}_i$ and $\vec{a}_j$.

We denote the vectors whose length is equal to the longest orthogonal projection of $\vec{d}_i, i=\overline{1,4}$ vectors to $\vec{a}_j$ vectors and whose direction is the same as the direction of $\vec{a}_j$ vectors as $\vec{b}_1^{'}$, $\vec{b}_2^{'}$, $\vec{b}_3^{'}$ respectively. We construct a parallelepiped $P^{'}$ whose edges consist of vectors $\vec{b}_j^{'}$, $j=\overline{1,3}$ and which contains the cube C^B. This will be $P^{'} \stackrel{*}{-} C^B \neq \varnothing$ according to the construction of this parallelepiped. As in the first case, to place the parallelepiped $P^{'}$ inside the cube C^A by parallel transfer, it is necessary and sufficient that the sides of $P^{'}$ are not greater than the corresponding sides of C^A, that is,

$$|\vec{a}_1| \geq |\vec{b}_1^{'}|,\ |\vec{a}_2| \geq |\vec{b}_2^{'}|,\ |\vec{a}_3| \geq |\vec{b}_3^{'}|, \tag{6}$$

thus

$$|\vec{a}_j| \geq |proj_{\vec{a}_j} \vec{d}_i|, i=\overline{1,4},\ j=\overline{1,3}. \tag{7}$$

With this we conclude the proof.

Key Words: Minkowski difference, orthogonal projection, scalar product.

REFERENCES

[1] M.Mamatov, J.Nuritdinov. Some Properties of the Sum and Geometric Differences of Minkowski. Journal of Applied Mathematics and Physics. 2020. Volume 8. pp.2241-2255.

[2] Nuritdinov J.T. On Minkowski difference of triangles. Bull. Inst. Math. Vol.4/6, 2021, pp. 50-57. [In Uzbek]

[3] Nuritdinov J.T. About the Minkowski difference of squares on a plane.// Scientific Reports of Bukhara State University. 2021. Volume 3. pp. 13-30.

On Matrixes of Comutative Quaternions

Hatice ÇETINKAYA[1] **and Gülay KORU YUCEKAYA**[2]
1 Selcuk Universty, Institute of Science, Department of Mathematics, Alaeddin Keykubat Campus, 42130, Selcuklu-Konya-Turkey,
hatice_barsbuga@hotmail.com
2 Selcuk University, Faculty of Sciences, Department of Mathematics, Alaeddin Keykubat Campus, 42130, Selcuklu-Konya-Turkey,
gulay.koru@selcuk.edu.tr

ABSTRACT

After W.R. Hamilton defined quaternions in 1843, C. Segre defined commutative quaternions in 1892. The generalized form of commutative quaternions are elliptical quaternions. A special case of elliptical quaternions are real quaternions. By defining a new multiplacition in the set of quaternions, Hamilton facilitated the study of motions in Euclidean space. In this study; the algebraic structure of commutative quaternions defined on the real numbers field is constructed and a 2x2 type matrix representation of this kind of quaternions is obtained.

Keywords: Field, Quaternion, Commutative Quaternion, Elliptic Quaternion, Commutative Quaternion Algebra

REFERENCES

[1] Vinberg, E.B., A Course in Algebra, Graduate Studies in Mathematics, American Mathematical Society, Providence, Rhade Island, 2003.

[2] Gallian, J.A., Contemporary Abstract Algebra, 2th Ed., Boston, 2013.

[3] Hacısalihoğlu, H. H., Motion Geometry and Quaternions Theory, Gazi University Faculty of Arts and Science Publications, Math. No.2, Ankara, 1983.

[4] Özdemir, M., Quaternions and Geometry, Golden Point Publishing, 1st Ed., Izmir, October 2020.

[5] Kösal, H.H., On Commutative Quaternion Matrices, Ph. D. Thesis, Sakarya University Institute of Science, p.147, Sakarya, 2016.

[6] Koru Yücekaya, G., Matrix Representation on Quaternion Algebra, Turkish Journal of Mathematics and Computer Science, Vol.2, Issue 1, p.1-7, 2014.

On Algebra of Elliptical Quaternions

Fadime ORMAN and Gülay KORU YUCEKAYA[2]
1 Selcuk Universty, Institute of Science, Department of Mathematics, Alaeddin Keykubat Campus,
42130, Selcuklu-Konya-Turkey,
fadimeaciyan123@gmail.com
2 Selcuk University, Faculty of Sciences, Department of Mathematics, Alaeddin Keykubat Campus, 42130, Selcuklu-Konya-Turkey,
gulay.koru@selcuk.edu.tr

ABSTRACT

Quaternions were developed in 1843 by W.R. It was defined by Hamilton. By defining a new product on the set of quaternions, Hamilton facilitated the study of motions in Euclidean space. In this study; algebraic structure of elliptic quaternions has been constructed on the field of real numbers and some properties of this kind of quaternions have been investigated.

Keywords: Field, Elliptic Quaternion, Elliptic Quaternion Algebra, Hamilton Operators

This study is a part of Fadime ORMAN's master's thesis.

REFERENCES

[1] Pierce, R.S., *Associative Algebras, Springer*, 1982.
[2] Hacısalihoğlu, H. H., Motion Geometry and Quaternions Theory, Gazi University Faculty of Arts and Science Publications, Math. No.2, Ankara, 1983.
[3] Özdemir, M., Quaternions and Geometry, Golden Point Publisınhing, 1st Ed., Izmir, October 2020.
[4] Agrawal, O.P., Hamilton Operators and Dual-Number-Quaternions in Spatial Kinematics. Mechanizm and Machine Theory, 22(6), p.569-675, 1987.
[5] Koru Yücekaya, G., Aktaş, K., Matrix Representation on Quaternion Algebra, ICAAA-2011, Istanbul.

CONSTRUCTING THE ELLİPSE AND ITS APPLICATIONS IN ANALYTICAL FUZZY PLANE GEOMETRY

Seçil Özekinci[1], Cansel Aycan [2]
1 Department of Mathematics, Pamukkale University, Denizli, 20070, Turkey
secilozizmirli@gmail.com

2 Department of Mathematics, Pamukkale University, Denizli, 20070, Turkey
aycancansel17@gmail.com

ABSTRACT

In this study, we talked about a method and the details of the method to be used to create the fuzzy ellipse. In the previously studies, some methods for fuzzy parabola are discussed.To define the fuzzy ellipse, it is necessary to modify the method applied for the fuzzy parabola. First, need to get five same points with the same membership grade to create crisp ellipse and the union of crisp ellipses passing through these points will form the fuzzy ellipse. Although it is difficult to determine the points with this property, it is important for constructing the fuzzy ellipse equation. In this study, we determine the points that satisfy this condition and prove the properties required to obtain the fuzzy ellipse to be formed by using these points. In the second part of this study, we have drawn a graph of a fuzzy ellipse and depicted the geometric location of fuzzy points with different membership grades on graph. We have also shown some geometric application on examples. It has been shown that the determinants defined in the calculation of the coefficients of the fuzzy ellipse using the maple program for different points and angles with the examples given, thus different fuzzy ellipses can be obtained. In addition, by investigating the membership of any point to the fuzzy ellipse, if the appropriate conditions are met, the equation of the ellipse passing through this point will be found. Then we can very easily obtain which one is the closest ellipse to the crisp ellipse.

Key Words: Fuzzy ellipse, fuzzy points, same fuzzy points, membership function, α- cuts.

REFERENCES

[1] L.A. Zadeh, Fuzzy Sets, Information and Control, vol 8, pp.338-353, 1965.

[2] J.J. Buckley, E. Eslami , “Fuzzy plane geometry I: points and lines”, Fuzzy Sets Syst., vol 86 , pp. 179–187, 1997

[3] D. Ghosh, D. Chakraborty , “Analytical fuzzy plane geometry I”, Fuzzy Sets Syst., vol 209, pp. 66–83 , 2012.

[4] J.J. Buckley, E. Eslami, “Fuzzy plane geometry II: circles and polygons”, Fuzzy Sets Syst., vol 87 , pp.79–85, 1997.

[5] D. Ghosh, D. Chakraborty, “Analytical fuzzy plane geometry II”, Fuzzy Sets Syst., vol 243, pp.84-109, 2013.

[6] D. Ghosh, D. Chakraborty, “Analytical fuzzy plane geometry III”, Fuzzy Sets Syst., vol 283, pp. 83-107, 2016.

[7] D. Ghosh, D. Chakraborty, An Introduction to Analytical Fuzzy Plane Geometry, Springer International Publishing, Cham, pp. 145-171, 2019.

TOPOLOGY

On a Topological Operator via Local Closure Function

Ayşe Nur TUNÇ [1], **Sena ÖZEN YILDIRIM** [2]
1,2 Çanakkale Onsekiz Mart University, Faculty of Science, Department of Mathematics, Çanakkale, Türkiye
aysenurtunc@comu.edu.tr, senaozen@comu.edu.tr

ABSTRACT

In general topology, there are important topological operators; such as closure, interior, boundary operators. After the concept of the ideal was introduced by Kuratowski in [2], many topological operators was studied in ideal topological spaces. For instance, local function [2], local closure function [1], $cl^\star$ Kuratowski closure operator [7], $\star$-boundary operator [6]. Among these topological operators, $cl^\star$ Kuratowski closure operator and $\star$-boundary operator were defined by using local function. In the paper [6], Selim et al. defined new closure operator by using $\star$-boundary operator and they researched some properties of this operator. Also, they characterized the Hayashi-Samuel spaces [8] by using $\star$-boundary points [6]. Furthermore, Al-Omari and Noiri were studied local closure function and they defined other operator ψ_Γ via local closure function in [1]. With the help of this operator, they formed the topologies σ_0 and σ which is coarser than σ_0. In [5], Goyal and Noorie defined θ-closure of a set with respect to an ideal by using local closure function and they obtained a new topology τ_{I_θ}. The authors in [1], [3], [4] studied the local closure function in detail.

In this study, we introduce new operator Bd^Γ by using local closure function in ideal topological spaces. We also investigate important properties of this operator.

Key Words: Operator Bd^Γ, local closure function, ideal topological space.

REFERENCES

[1] A. Al-Omari and T. Noiri, Local Closure Functions in Ideal Topological Spaces, Novi Sad J. Math. 43(2) (2013), 139-149.

[2] K. Kuratowski, Topology, Vol. I, Academic Press, New York, 1966.

[3] N. S. Noorie and N. Goyal, On $S_{2\frac{1}{2}}$ mod I Spaces and θ^I-closed Sets, International Journal of Mathematics Trends and Technology 52(4) (2017), 226-228.

[4] A. Pavlović, Local Function Versus Local Closure Function in Ideal Topological Spaces, Filomat 30(14) (2016), 3725-3731.

[5] N. Goyal and N. S. Noorie, θ-closure and $T_{2\frac{1}{2}}$ Spaces via Ideals, Italian Journal of Pure and Applied Mathematics 41 (2019), 571-583.

[6] Sk Selim, S. Modak and Md. M. Islam, Characterizations of Hayashi-Samuel Spaces via Boundary Spaces, Commun. Adv. Math. Sci. 2(3) (2019), 219-226.

[7] R. Vaidyanathaswamy, The Localisation Theory in Set-Topology, Proc. Indian Acad. Sci., Sect. A. 20 (1944), 51–61.

[8] J. Dontchev. Idealization of Ganster-Reilly Decomposition Theorems, arXiv: math. Gn/9901017v1, 5 Jan 1999.

Coverings of Hom-Lie Crossed Modules

Tunçar Şahan [1] and Ali Aytekin [2]
[1] Department of Mathematics, Aksaray University,
tuncarsahan@aksaray.edu.tr
[2] Department of Mathematics, Pamukkale University,
aaytekin@pau.edu.tr

ABSTRACT

Crossed modules on groups were defined by Whitehead [1]. After that, they are defined on many algebraic structures such as algebra, commutative algebra, Lie algebra, Leibniz algebra, Lie-Rinehart algebra and their properties, and their relationship with other structures has been investigated [2, 3]. Recently, crossed modules over Hom-Lie algebras were introduced by Shen and Chen in [4] in order to reveal relationship between Hom-Lie 2-algebras and crossed modules on Hom-Lie algebras. After then, many authors studied this topic. Especially, Casas and Garcia-Martinez investigated crossed modules of Hom-Lie algebras in [5], and they studied on low dimensional cohomology groups of Hom-Lie algebra and their relation with crossed modules. Also, they defined cat^1-Hom-Lie algebras and showed natural equivalence between category of cat^1-Hom-Lie algebras and category of crossed modules over Hom-Lie algebras.

Covering for crossed modules was first defined by Brown and Mucuk in [6] using the equivalence of crossed modules and group-groupoids. The concept of coverings for groupoids were obtained using fundamental groupoid functor (π_1) by Brown, Danesh-Naruie and Hardy [7]. In the same study, they defined coverings for internal groupoids in any category with pullback structures. With similar thinking, in this study, we will obtain coverings of Hom-Lie crossed modules and give some their properties.

Key Words: Hom-Lie algebra, crossed module, covering.

REFERENCES

[1] Whitehead JHC. Combinatorial homotopy. Bulletin of the American Mathematical Society (1949), 55: 453-496.

[2] Arvasi Z. Crossed modules of algebras. Mathematical Computational Applications (2004), 9 (2): 173-182.

[3] Arvasi Z, Odabaş A. Computing 2 dimensional algebras Crossed modules and Cat^1 sup algebras. Journal of Algebra and Its Applications (2016), 15 (10): 165-185.

[4] Sheng Y, Chen D. Hom-Lie 2-algebras. Journal of Algebra (2013), 376: 174-195.

[5] Casas JM, García-Martínez X. Abelian extensions and crossed modules of Hom-Lie algebras. Journal of Pure and Applied Algebra (2020), 224: 987-1008.

[6] Brown, R., Mucuk, O., Covering groups of non-connected topological groups revisited, Math. Proc. Camb. Phil. Soc. (1994), 115, 97–110.

[7] Brown, R., Danesh-Naruie, G., Hardy, J.P.L., Topological Groupoids: II. Covering Morphisms and G-Spaces, Math. Nachr. (1976), 74, 143–156.

A note on quasi-metrizable spaces

Hülya Duru[1], Serkan ilter [2] and Aygül Bilgin[3]
1 Istanbul University, Faculty of Science, Istanbul, Turkey,
hduru@istanbul.edu.tr
2 Istanbul University, Faculty of Science, Istanbul, Turkey,
ilters@istanbul.edu.tr
3 Haliç University, Department of Mathematics, Istanbul, Turkey,
aygulbilgin@halic.edu.tr

ABSTRACT

Let X be a non-empty set and A, B two subsets of $X \times X$. By definition,

$A^{-1} = \{(a,b):\ (b,a) \in A\}$, $t \in [0,T]$,

and

$A \circ B = \{(a,c):\ \text{there exists } b \in X \text{ such that } (a,b) \in B \text{ and} (b,c) \in A\}$.

A filter $\mathcal{U}_q$ on $X \times X$ is called a quasi-uniformity if each element of $\mathcal{U}_q$ contains the diagonal set and for each $W \in \mathcal{U}_q$ there exists $V \in \mathcal{U}_q$ satisfying $V \circ V \subseteq W$.

It is well know that a quasi-uniformity generates a topology. The converse, that is, there is a quasi-uniformity for a given topology compatible with the topology, was first proved by Krishnan [2], then Császár proved it with the structure he defined [1]. But the direct topological proof was given by Pervin [3].

A subcollection of a given quasi-uniformity is a basis for the quasi-uniformity if each element of the quasi-uniformity includes at least one element of the subcollection in question. A function $d: X \times X \to [0,\infty[$ is said to be a quasi-metric if $d(x,x)=0$ and $d(x,z) \le d(x,y)+d(y,z)$ ($x,y,z \in X$). In this case, X is said to be quasi-metrizable if the family $(B_\lambda(a))_{\lambda>0}$ is a local basis at each $a \in X$ where $B_\lambda(a)$ is the set containing all points $b \in X$ satisfying $d(a,b) < \lambda$.

In this talk, we discuss the conditions to obtain a quasi-metrizable topological space.

Key Words: quasi-uniformity, quasi-metric

REFERENCES

[1] Császár, Fondements de la topologie générale, Akadémiai Kiadó, Budapest, 1960

[2] V. S. Krishnan, A Note on Semi-Uniform Spaces, J. Madras Univ. Sect. B, 25 (1955) , 123-124

[3] W. J. Pervin, Quasi-Uniformization of Topological Spaces, Math. Annalen 147 (1962), 316-317.

A note on global classical solutions to a Cauchy problem

Serkan ilter [1], Hülya Duru[2] and Havva Nur Öztürk[3]
1 Istanbul University, Faculty of Science, Istanbul, Turkey, ilters@istanbul.edu.tr
2 Istanbul University, Faculty of Science, Istanbul, Turkey, hduru@istanbul.edu.tr
3 Istanbul University, Institute of Science, Istanbul, Turkey, havvanurozturk04@gmail.com

ABSTRACT

Let $F:[0,T]\times R^n \to R^n$ be a set-valued map with nonempty values. In this paper, we study the existence of global classical solutions to the following problem, called a Cauchy problem such that $x'(0)=y_0$, for every $x_0 \in R^n$ and $y_0 \in F(0,x_0)$,

$$x'(t) \in F(t,x(t)),\ t \in [0,T],$$

$$x(0)=x_0,\ x'(0)=y_0.$$

The existence of local (or global) classical solutions in the nonconvex (or convex) case for this problem, under the assumption that each set F(t,x) is uniformly locally connected (or totally disconnected), was studied in many papers (see, e.g. [1], [2]). For this purpose, we make use of some results related to the existence of Caratheodory type solutions under different assumptions.

Key Words: differential inclusion, existence, classical solution

REFERENCES

[1] Bressan and Z.P. Wang, Classical solutions to differential inclusions with totally disconnected right-hand side. J. Differ. Equ. 246 (2009) 629–640

[2] A.F. Filippov, The existence of solutions of generalized differential equations, Mathematical notes of the Academy of Sciences of the USSR volume (translation of Matematicheskie Zametki) 10 (1971), 608-611.

[3] J. P. Aubin and A. Cellina, Differential Inclusions, *Springer-Verlag, Berlin*, 1984.

Some Properties of Metric in Digital Topology

Gulseli BURAK [1]
1 Pamukkale University, Faculty of Science, Department of Mathematics, Denizli, Türkiye.
germez@pau.edu.tr

ABSTRACT

The aim of this study is consider some properties of a digital image. A digital image *X* is a subset of $\mathbb{Z}^n$ for some natural number *n* together with an adjacency relation κ inherit from $\mathbb{Z}^n$ and represented by (X, κ). We analyze digital images by using the shortest digital simple path metric which is denoted by d^k. We use digital topological concepts such as adjacency relation, the *r-thickening* of the subsets of a connected digital image and *digital Hausdorff distance* to explore the properties of digital images with the shortest digital simple path metric. The *r-thickening* of a subset is defined by taking the union of neigborhoods with radius *r* of all points in subset. The *digital Hausdorff distance* between two noempty subsets *A* and *B* of κ-connected digital image *X* is defined minimum *r* such that the *r-thickening* of each subset will contain another.

In this study we define distance of two subsets of a connected digital image via the shortest simple path metric. We examined digital topological versions of metric-related features using the shortest simple path metric. Applications on various subsets of digital images are explored. We give examples showing that when the adjacency relation defined on a digital image changes, the distance between the points taken on the image changes.

Key Words: Digital topology, digital image, Hausdorff metric.

REFERENCES

[1] L. Boxer, Multivalued functions in digital topology, arXiv:1703.01700v1 (2017).

[2] L. Boxer, Beyond the Hausdorff metric in digital topology, arXiv: 2108.03114v2 (2021).

[3] T. Vergili, Digital Hausdorff distance on a connected digital image, Commun. Fac. Sci. Univ. Ank. Ser. Al Math. Stat. 69:2(2020), 1070-1082.

Banach and Kannan Type Contracting Mappings for Box Metric in Digital Images

Simge ÖZTUNÇ [1], Aysun ERDOĞAN SERT [2]
1 Manisa Celal Bayar University, Faculty of Science and Arts, Department of Mathematics, Prof. Dr. İlhan Varank Campus, 45030, Yunus Emre Manisa/TURKEY, e-mail: simge.oztunc@cbu.edu.tr
2 Manisa Celal Bayar University, Faculty of Science and Arts, Department of Mathematics, Prof. Dr. İlhan Varank Campus, 45030, Yunus Emre Manisa/TURKEY, e-mail: aysn_erdogn@hotmail.com

ABSTRACT

The notion of digital topology is a field that uses geometric and algebraic topology to examine the geometric and topological properties of digital images. Digital topology has applications used in computing areas such as image processing and computer graphics. Digital topology is a field of mathematics that develops by using the topological properties of objects, and more by using the characteristics of two and three-dimensional digital images.

On the other hand, fixed point theory is one of the important and developing areas of mathematics. One of the most fundamental theorems in fixed point theory is Banach fixed point theorem. The reason why this theorem is important is that it has a wide range of applications and has created fundamentals for proving different fixed point theorems. One of the important theorems proven after the Banach fixed point theorem is Kannan fixed point theorem. Kannan fixed point theorem is a theorem that is still being studied in various metric spaces and its applications are presented. The applications of fixed point theory are used in many scientific areas.

In this study, firstly, preliminaries about digital topology, fixed point theory and box metric are given. The properties that distinguish digital metric spaces from metric spaces are examined. The most prominent of these properties is that the ε is taken as 1 because it is studying on a finite subset of $\mathbb{Z}^n$. This property has played an important role in defining basic concepts such as convergence sequence and Cauchy sequence in digital metric spaces. In digital images, the Banach fixed point theory is then proved by defining a special metric called box metric, considering this metric in digital images with $\kappa-$adjacency relation and using the properties of digital metric space. Then, Kannan fixed point theorem is given by expressing Kannan type digital contraction mapping in digital box metric spaces.

Key Words: Digital box metric, Banach fixed point theory, Kannan type digital contraction mapping.

REFERENCES

[1] S. Banach, Sur les operations dans les ensembles abstraits et leur application aux equations integrales, Fund Math. 3 (1922), 133 181.
[2] L. Boxer, Digitally Continuous Functions, Pattern Recognition Letters, 15 (1994), 833-839.
[3] E. Deza, M. M. Deza, Encylopedia of distances, Springer Berlin Heidelberg (2009)

[4] O. Ege ve I. Karaca, Banach Fixed Point Theorem for Digital Images, J. Nonlinear Sci. Appl., 8 (2015),237-245.
[5] O. Ege, I. Karaca, Lefschetz Fixed Point Theorem for Digital Images, Fixed Point Theory Appl., 2013 (2013), 13 pages.
[6] S. E. Han, Banach Fixed Point Theorem from the Viewpoint of Digital Topology, J. Nonlinear Sci. Appl., 9 (2015), 895-905.
[7] R. Kannan, Some Results on Fixed Points, Bull. Calcutta Math. Soc., 60 (1968), 71-76.
[8] A. Rosenfeld, Digital Topology, Amer. Math. Monthly, 86 (1979), 76-87.

FUNDAMENTALS OF MATHEMATICS AND MATHEMATICS LOGIC

Directed Orbital Triangle Graphs

Murat Beşenk [1], Kübra Oymak[2]
1 Pamukkale University, Faculty of Sciences, Department of Mathematics, mbesenk@pau.edu.tr
2 Pamukkale University, Faculty of Sciences, Department of Mathematics, kubratuksal@gmail.com

ABSTRACT

It is known that when graph topics are investigated, it is seen that there is a lot of study in literature. One of them is suborbital graphs. In [1], the connection between transitive groups and graphs was introduced and used to give new insight into some known results. In [2] and [4] were examined some properties of suborbital graphs for the group $\Gamma_0(N)$. They first introduced an invariant equivalence relation by using the congruence subgroup $\Gamma_0(N)$ and obtain some results for the newly constructed subgraphs $F_{u,N}$ whose vertices form the block infinity. They obtained edge and circuit conditions and some relations between lengths of circuits in $F_{u,N}$ and elliptic elements of $\Gamma_0(N)$. And also the structures of directed and undirected graphs were analysed and applied to primitive groups. On the other hand, the signature of a discrete group consists of the geometric invariants. The signature on the working group is extremely important in terms of revealing invariants. The main purpose in this study, is to set the foundations of a new method which would help to identify any discrete group much better. Therefore, because of that the signature problem is transferred to the suborbital graphs and a new approach is tried to be achieved.

Key Words: Suborbital graphs, imprimitive action, circuit.

REFERENCES

[1] C.C. Sims, Graphs and Finite Permutation Groups, Mathematische Zeitschrift 95 (1967),76-86.

[2] M. Akbaş, On Suborbital Graphs For The Modular Group, Bulletin London Mathematical Society 33 (2001), 647-652.

[3] N. L. Biggs, A. T. White, Permutation Groups and Combinatorial Structures, Cambridge University Press, Cambridge, 1979.

[4] G. A. Jones, D. Singerman, K. Wicks, The Modular Group and Generalized Farey Graphs, Bulletin London Mathematical Society 160 (1991), 316-338.

[5] M. Beşenk, B. Ö. Güler, A. Büyükkaya, Suborbital Graphs For a Non-Transitive Action of The Normalizer, Filomat 33 (2019) 385-392.

$K_{1,1}$-Structure Connectivity of Generalized and Double Generalized Petersen Graphs

Gülnaz Boruzanlı Ekinci [1]
1 Department of Mathematics, Faculty of Science, Ege University, İzmir, Türkiye
gulnaz.boruzanli@ege.edu.tr

ABSTRACT

The connectivity notion is an important measure for the reliability of graphs, and it gives the cardinality of a minimum vertex–cut. Since determining the reliability and fault tolerance of a graph becomes an important issue, several variants of the classical connectivity notion have been defined and studied in the literature. However, most of these studies have focused on the effect of individual vertices becoming faulty. In most real-world problems, adjacent vertices in a network might affect each other and the neighbours of a faulty vertex might be more vulnerable. Motivated also by the fact that networks and subnetworks are made into chips in today's technology, Lin et al. [1] introduced a new kind of connectivity parameter called structure connectivity. According to this definition, a set F of connected subgraphs of G is a subgraph cut of G if G − V(F) is disconnected or a trivial graph. Let H be a connected subgraph of G. Then F is an H-structure-cut if F is a subgraph cut, and every element of F is isomorphic to H. The H-structure connectivity of G is the minimum cardinality of all H-structure-cuts of G. Similarly, F is an H-substructure-cut if F is a subgraph-cut, such that every element of F is isomorphic to a connected subgraph of H. The H-substructure-connectivity of G is the minimum cardinality of all H-substructure-cuts of G.

The class of generalized Petersen graphs was introduced by Coxeter [2] and named by Watkins [3]. Later, Zhou and Feng [4] introduced the notion of double generalized Petersen graphs to classify cubic vertex-transitive non-Cayley graphs of order 8p, for any prime p. In this study, we investigate the fault tolerance of these two classes, namely generalized and double generalized Petersen graphs, in terms of $K_{1,1}$-structure connectivity and $K_{1,1}$-substructure connectivity. The obtained results are of practical interest since graphs in these classes are good candidates for

interconnection networks due to their desired properties such as regularity, small girth, super-connectedness [5], and Hamiltonicity [6,7].

Key Words: structure connectivity, generalized Petersen graphs, fault-tolerance.

REFERENCES

[1] C.K. Lin, L. Zhang, J. Fan and D. Wang, Structure connectivity and substructure connectivity of hypercubes, Theoretical Computer Science 634 (2016), 97-107.

[2] H. S. M. Coxeter, Self-dual configurations and regular graphs, Bull. Amer. Math. Soc. 56(5) (1950), 413-455.

[3] M. E. Watkins, A theorem on tait colorings with an application to the generalized Petersen graphs, Journal of Combinatorial Theory 6(2) (1969), 152-164.

[4] J.-X. Zhou and Y.-Q. Feng, Cubic vertex-transitive non-Cayley graphs of order 8p, The Electronic Journal of Combinatorics 19(1) (2012) 53-53.

[5] G. Boruzanlı Ekinci and J.B. Gauci, On the reliability of generalized Petersen graphs, Discrete Applied Mathematics 252 (2019), 2-9.

[6] B. Alspach, The classification of hamiltonian generalized Petersen graphs, Journal of Combinatorial Theory Series B 34(3) (1983), 293-312.

[7] X. Wang, All double generalized Petersen graphs are Hamiltonian, Discrete Mathematics 340(12) (2017), 3016-3019.

On Some Basic Properties Of Finite Graph Theory

Ali Gökhan Ertaş [1] and <u>İbrahim Günaltılı</u>[2]
1 Kütahya Dumlupınar University, aligokhan.ertas@dpu.edu.tr
2 Eskişehir Osmangazi University,Science Faculty, igunalti@ogu.edu.tr

ABSTRACT

The graph theory can facilitate the solution by transforming complex structures and expressions into a more understandable model. It has found a place in the modern world as it enables the definition of problems and the determination of their structural relations.

The objective of the this paper is to introduce participants with the fundamental concepts in graph Theory, with a sense of some its basic properties. In the light of basic graph definition, finite graph, multiple graph, pseudo graph, complete graph, bipartite graph, cycle graph, path graph, tree graph , regular graphs and their examples will be explained.

Finally, brief information will be given about the circular graphs defined for the first time in the literature by İbrahim Günaltılı.

Key Words: Finite graph , complete graph, bipartite graph

REFERENCES

[1] Günaltılı,İ.,Kurtuluş,A.: «On Finite Near-Circular Spaces» , Applied Sciences, cilt.6, ss.21-24, 2004

[2] Sunar, R. , Günaltılı, İ. "On The Basic Properties of Linear Graphs - I" . Konuralp Journal of Mathematics 9 (2021),154-158

[3] Günaltılı,İ : «Finite Reguler {0,1}-bigraphs», 2nd Cyprus International Conference on Educational Research (CY-ICER), Cyprus, 14 - 16 Şubat 2013, cilt.89, ss.529-532

[4] Chartrand,G.,Lesniak,L.,Zhang,p. : «Graphs and Digraphas»,Crc Press fifth edition,Isbn-10 1439826277 ,2010

On Projective Graphs

Ramazan Sunar[1] and İbrahim Günaltılı[2]
Ministry National Education, ramazan_sunar03@hotmail.com
Eskişehir Osmangazi University, igunalti@ogu.edu.tr

ABSTRACT

H. M Mulder introduced $(0,\lambda)-$ graph. A $(0,\lambda)-$ graph is a connected graph in which each pair of vertices have λ common neighbours or none at all. ($\lambda \geq 2$). In the following years, some studies were done for case $\lambda = 1$ under Günaltılı's direction in [4]. Thus, some basic properties of $(0,1)-$ graphs were examined and some classifications were made.

A linear graph is a bipartite $G = (P \cup L, E)$ with the properties;

i. For all $u, v \in P$ such that $u \neq v$, $cn(u,v) = 1$.

ii. $\delta(G) \geq 2$.

The basic properties of linear graphs are reviewed in [6,7].

A projective graph is a linear graph $G = (P \cup L, E)$ with the properties

i. For all $X, Y \in L$ such that $X \neq Y$, $cn(X,Y) = 1$.

ii. $C_8 \subset$ G.

In this study, first of all, the basic properties of projective graphs such as the number of vertices and edges, diameter and regularity were examined. By examining the necessary parameters for linear graphs to be projective graphs, $(n+1)-$ regular linear graphs are characterized. In addition, whether the projective graphs are Euler graphs or not has been examined. In the last part of the study, some properties of the approximate projective graphs obtained by subtracting the second condition given in the projective graph are examined.

Key Words: Linear graph, $(0,1)-$ graph, common neighbour.

REFERENCES

[1] E. Brouwer, Classification of small (0,2)-graphs, J. Comb. Theory, Ser., (2006), A 113, 1636–1645.

[2] D. B. West, Introduction to graph theory, Upper Saddle River: Prentice hall, 2, (2001).

[3] H .M. Mulder, (0, λ)-graphs and n-cubes, Discrete mathematics, (1979), 28(2): 179–188.

[4] İ. Günaltılı, A. Ulukan and Ş. Olgun, Some properties of finite {0,1}-graphs, Konuralp Journal of Mathematics, (2013) 1(1): 34-39.

[5] L.M. Batten, Combinatorics of finite geometries, 2nd ed. English, Cambridge University Press, 1997.

[6] R. Sunar, İ., Günaltılı, On The Basic Properties of Linear Graphs-I, Konuralp Journal of Mathematics (KJM), (2021), 9(1): 154-158.

[7] R. Sunar, Lineer Graflar Üzerine, Doktora Tezi, Eskişehir Osmangazi Üniversitesi, Fen Bilimleri Enstitüsü, 2021.

MATHEMATICS EDUCATION

Adaptive technologies for 21st century mathematics education

Péter Négyesi [1]
1 Eszterházy Károly Catholic University, Doctoral School of Education, 3300 Eger Klapka György str. 12., Hungary, negyesi.peter@uni-eszterhazy.hu

ABSTRACT

Adaptive learning is an emerging technology in education that teachers use to teach students and assess their performance (Smith, 2016). Adaptive technology has great potential because it gives teachers the opportunity to differentiate based on individual needs (Bilous, 2019). Its effectiveness lies in the fact that it provides immediate feedback to learners and can help to enhance the learning experience (Matherson & Windle, 2017).

Like many other technologies, it's constantly evolving as well, which makes it difficult to research and update curriculum as programs are constantly changing (Parsons, 2014). Teachers must also keep up with these changes. Instead of lecture-based learning, students need to be given more autonomy and teachers are expected to deliver interactive, engaging and effective lessons (Matherson & Windle, 2017).

Since little empirical research has specifically addressed adaptive technology and its impact on mathematics education in the 21st century, the purpose of this paper is to provide a snapshot of adaptive technology, the benefits of its use, and the current challenges teachers and students face when using the technology.

School leaders believe in investing in technology; however, research shows that simply investing in software and smart devices is not enough (Zhu, Yu, & Riezebos, 2016). Teachers also need to know how to use the tools effectively. Students need to embrace the concept of computer-assisted learning.

The results of this study suggest that the use of adaptive technologies is the cornerstone of experiential mathematics education in the 21st century, aimed at increasing students' achievement, motivation, and positive attitudes toward mathematics learning.

Keywords: Adaptive technologies, mathematics education, challenges.

REFERENCES

[1] Bilous, V. (2019). Basic principles for developing an adaptive learning system. Open Educational E-Environment of Modern University, Special Edition, 23-31. https://doi.org/10.28925/2414-0325.2019s3

[2] Matherson, L., & Windle, T. M. (2017). What do teachers want from their professional development? Four emerging themes. Delta Kappa Gamma Bulletin, 83(3), 28.

[3] Parsons, D. (2014). A mobile learning overview by timeline and mind map. International Journal of Mobile and Blended Learning, 6(4), 1-20. https://doi.org/10.4018/ijmbl.2014100101

[4] Smith, P. (2016). Personalized education using adaptive learning technology: One size. Learning, 12(1), 101-118.

[5] Zhu, Z.-T., Yu, M.-H., & Riezebos, P. (2016). A research framework of smart education. Smart Learning Environments, 3(1). https://doi.org/10.1186/s40561-016-0026-2

Current Strategies for the Empowerment of Mathematics Teachers

Yeliz ÖZKAN HIDIROĞLU [1]
1 National Ministry of Education, Denizli, Mathematics Teacher, yelizozkan09@gmail.com

ABSTRACT

Mathematics score of averages of Turkey in national and international exams are less than expected. In the tests conducted by OECD within the framework of the PISA program, its scores ranged between 420 and 454 between the years of 2003 and 2018. In 2015, Turkey experienced a dramatic decrease with 420 points. Besides, the averages of total mathematics scores in LGS (High School Entrance Exam) and OSS (Student Selection and Placement Test) are also quite low. This is also a sign that mathematics learning level in classrooms is low. In this case, it is of critical importance to build rich learning environments, especially in schools, where mathematics-based skills can be developed. This can be ensured through the empowerment of mathematics teachers who are the main actors of designing the desired learning environments. Since the technological and scientific developments of each age are different, the teacher profiles needed also differ depending on the age. In order to raise individuals that will be needed in the future with the 21st century skills, the competencies of teachers who will equip them with these skills stand out as an important research topic. Developing essential 21st century skills in the classroom environment including the critical thinking, communication, collaboration, creativity, various literacy skills (digital literacy, technology literacy, financial literacy, media literacy etc.), computational thinking, mathematical modelling, metacognitive thinking, interdisciplinary thinking, leads to changes in the roles of mathematics teachers in the learning process. In this sense, the aim of the study is to reveal current strategies for the empowerment of mathematics teachers and to focus on teacher empowerment strategies that can be followed in the future by critically considering different perspectives in the literature on the empowerment of mathematics teachers. The study is a theoretical review study.

Key Words: Teacher empowerment, mathematics teacher, teacher empowerment strategies.

REFERENCES

[1] Blase, J., & Blase, J.R. (1994). *Empowering teachers: What successful principals do*. Thousand Oaks, CA: Corwin Press.

[2] Özkan Hıdıroğlu, Y. & Tanrıöğen, A. (2020). Development of teachers' empowerment scale: A validity andreliability study. *International Journal of Assessment Tools in Education, 7*(4), 753-772.

[3] Özkan Hıdıroğlu, Y. (2021). *Öğretmenlerin güçlendirilmesinin örgütsel bağlılık, mesleki bağlılık ve örgütsel vatandaşlık üzerindeki etkisi.* Yayımlanmamış yüksek lisans tezi, Pamukkale Üniversitesi, Eğitim Bilimleri Enstitüsü, Denizli.

[4] Özkan Hıdıroğlu, Y. & Tanrıöğen, A. (2020). Thematic, methodological and statistical analysis of research on teacher empowerment. *Inonu University Journal of the Faculty of Education, 21*(2), 616-635.

[5] Thomas, S. (2017). Teacher empowerment: A focused ethnographic study in Brunei Darussalam. *The Qualitative Report, 22*(1), 47-72.

Mathematical Reasoning and Proof in Fifth Grade Mathematics Textbook

Sare Şengül[1], Büşra Kıral[2]
1 Marmara University, Faculty of Education, Department of Mathematics and Science Education, Istanbul, Turkey, zsengul@marmara.edu.tr
2 Istanbul Aydin University, Faculty of Education, Department of Mathematics and Science Education, Istanbul, Turkey, busrakiral@aydin.edu.tr

ABSTRACT

Science confirms by observation, mathematics by reasoning, and the essence of mathematics lies in proofs. Creating valid arguments or proofs and criticizing arguments are integral parts of doing math. Mathematics is a science based on concepts and operations with a certain order and logical order. To discover and make sense of this order is to do mathematics. For this reason, it can be said that school mathematics should provide students with opportunities to explore and make sense of it. In order for students to take advantage of these opportunities, we can say that mathematics learning environments should provide students with mathematical reasoning and proof skills. Analyzing the reasoning and proving activities in textbooks, one of the factors that affect mathematics teaching and learning, is the first important step to understanding the opportunities for students to learn to reason and prove. This research examines a fifth-grade mathematics textbook presented to all students by the Ministry of National Education through the Education Informatics Network in the context of activities for mathematical reasoning and proof. In the analysis of the mathematics textbook, the analytical framework developed by the researchers based on the evaluation criteria in the studies of Stylianides (2008, 2009) and Thompson et al. (2012) is used for data analysis. In this context, reasoning and proof activities are evaluated separately according to the secondary school fifth-grade mathematics learning areas, the activities in the chapters of the book and the purposes of the activities. The analysis process continues according to the purposes of reasoning and proof activities and the types of arguments the students can create.

Key Words: Reasoning and proof, mathematics, textbook analysis.

REFERENCES

[1] Hong, D. S., & Choi, K. M. (2018). Reasoning and Proving Opportunities in Textbooks: A Comparative Analysis. *International Journal of Research in Education and Science*, *4*(1), 82-97.

[2] Ross, K. A. (1998). Doing and proving: The place of algorithms and proofs in school mathematics. *The American Mathematical Monthly*, *105*(3), 252-255.

[3] Stylianides, G. J. (2008). An analytic framework of reasoning-and-proving. *For the learning of mathematics*, *28*(1), 9-16.

[4] Stylianides, G. J. (2009). Reasoning-and-proving in school mathematics textbooks. *Mathematical thinking and learning*, *11*(4), 258-288.

[5] Thompson, D. R., Senk, S. L., & Johnson, G. J. (2012). Opportunities to learn reasoning and proof in high school mathematics textbooks. *Journal for Research in Mathematics Education*, *43*(3), 253-295.

[6] Van De Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2012). *İlkokul ve ortaokul matematiği: Gelişimsel yaklaşımla öğretim.* (Çev. Editörü: Soner Durmuş). Ankara: Nobel Yayın Dağıtım.

Application of Virtual Manipulatives and Simulations to Mathematics Lesson Plans

Güler Tuluk[1], Özgenur Yılmaz[2],
Kastamonu University Education Faculty, gtuluk@gmail.com
Mathematics Teacher, MoNe, ozge.yilmaz.1996@gmail.com

ABSTRACT

Developing technology and changing educational understandings cause rapid increases in the introduction of technology into education. When the concept of technology is considered, tools usually come to mind (eg computer, CD, DVD), the meaning of technology is much broader. Instead of seeing technology as a "side job" in classroom activities, it should be considered as a complement to learning tools used in education. In this study, it is aimed to use the blended learning approach in which web-assisted and face-to-face learning are carried out together in mathematics teaching and to prepare lesson plans suitable for this approach. The stages of integrating virtual manipulative and simulations, which are computer applications in web-assisted learning, into face-to-face learning are discussed in detail within the framework of preparing a lesson plan. This learning offers the possibilities of Web-assisted learning along with student-student and student-teacher interaction in the teaching-learning process. This approach is considered important as it will add diversity to learning and can be one of the approaches that can be applied in the future. Within the scope of the study, virtual manipulative and simulation sites were determined that are suitable for mathematics acquisitions using web 2.0 tools. By including the use of the applications on these sites in the classroom environment in the lesson plans, it will be ensured that the teaching is planned and systematic. In this study, examples were given to teachers about introducing virtual manipulative and simulation sites and adapting these applications to mathematics lesson plans. There are advantages such as richness of learning, access to information, social interaction, and management of learning in the plans in which both face-to-face and web-supported applications are used together in teaching.

Key Words: Virtual Manipulative and simulations, math teaching, lesson plan

References

[1] Curcio, I. D., Dipace, A., & Norlund, A. (2016). Virtual realities and education. Research on Education and Media, 8(2), 60-68.

6th Grade Students' Construction Processes of Prime Number Concept in Realistic Mathematics Education Based Environment

Nehir Keyik[1] and Rezan Yilmaz [2]
1 Ondokuz Mayis University, Graduate School of Educational Sciences, nehirkeyik@gmail.com
2 Ondokuz Mayis University, Faculty of Education, rezzany@omu.edu.tr

ABSTRACT

The purpose of this study is to research 6th grade students' construction processes of prime number concept in the teaching environment designed in accordance with the Realistic Mathematics Education (RME). The study was conducted in a classroom with 15 students in a secondary school in the Black Sea Region and it was designed as a case study, one of the qualitative research methods. A readiness test was prepared and implemented in order to measure the students' preliminary knowledge about the concept of prime number. The students were divided into heterogeneous groups according to the results of the readiness test, observations of one of the researcher and the information obtained from their math teacher. Students were asked to solve a contextual problem posed in the learning environment of RME, within the group. The teaching process was supported by in-group and intergroup discussions. Clinical interviews were held with three participants who were selected from the groups by purposeful sampling and whose readiness levels were good, intermediate and pre-intermediate. The data obtained from the transcripts of the camera and audio recordings of the teaching process, in-group discussions and interviews, individual and group worksheets and observations were analyzed within the framework of APOS Theory. APOS Theory is a theory of how mathematical concepts can be learned and focuses on models of what might be going on in the mind of an individual when he/she is trying to learn a mathematical concept and uses these models to design instructional materials and/or to evaluate student successes and failures in dealing with mathematical problem situations. (Arnon et al., 2014). Content analysis, one of the qualitative research methods, was used in the research. The main purpose in content analysis is to reach concepts and relationships that can explain the collected data. (Yıldırım and Şimşek, 2016).

As a result of the study, three participants conceptualized the prime number as an object. In addition, it has been determined that the coordination of the concepts of divisor, multiple, divisibility rules, odd-even number and factor tree are very important in the formation of the concept of prime number. According to the results of the research, suggestions for future researches in teaching the concept of prime number are also presented.

Key Words: prime number, APOS Theory, concept formation, abstraction realistic mathematics education.

REFERENCES

[1] Arnon, I., Cottrill, J., Dubinsky, E., Oktaç, A., Fuentes, S. R., Trigueros, M. ve Weller K. (2014). *APOS Theory: A Framework for Research and Curriculum Development in Mathematics Education*. New York: Springer.

[2] Yıldırım, A. & Şimşek, H. (2016). *Sosyal Bilimlerde Nitel Araştırma Yöntemleri* (10. baskı). *(Qualitative Research Methods in Social Sciences- 10th ed.),* Ankara: Seçkin Yayıncılık

[3] Farrell, T. S. (2002). Lesson planning. *Methodology in language teaching: An anthology of current practice*, *11*(2), 30-39.

The Investigation of SSCI Indexed Studies on Problem Posing in Terms of Affective Components

Gülşah GEREZ CANTİMER [1] **and Sare ŞENGÜL**[2]
1 Ministry of Education, Sakarya/Serdivan IHO, gulsahgerez@subu.edu.tr
2 Marmara University, zsengul@marmara.edu.tr

ABSTRACT

The problem posing activities play an important role in students' understanding of mathematical concepts [1]. It has a strategic position in the teaching and learning of mathematics, as it encourages creative thinking and enables the use of existing knowledge [2]. Considering that students' affective characteristics shape mathematics learning outcomes, especially mathematics achievement [3], the need for a holistic analysis of studies that deal with problem posing studies in terms of affective components arises.

In this direction, the aim of the research is to examine the general characteristics, aims and results of the studies published in SSCI indexed journals for problem posing in terms of affective components, through a systematic review. Thus, unlike other problem posing studies, the general framework of the related researches, which includes problem posing and affective components, from their aims to results will be drawn and their tendencies will be described and evaluated. The Web of Science database was used in the study and the keywords were determined as "problem posing, mathematics, attitude, self-efficacy, affective factors, motivation".

According to the research findings, 19 journal/book/book chapters scanned in the Web of Science where the studies were published were reached. A total of 24 studies, including 15 articles, 7 papers and 2 book/book chapters were examined from the sources reached. More than half of the studies were published between 2018-2020. It has been determined that a small number of studies have been carried out including one study between 2006-2008, 2012-2014 and 2021-2022. Affective areas examined together with problem posing; it has been determined that attitude (f=10) is mostly studied on "attitude, self-efficacy, motivation, anxiety and affective factors", while self-efficacy (f=6) and motivation (f=4) follow the attitude studies

quantitatively. The distribution of the studies according to the aims and results is examined in detail and suggestions will be made according to the outputs obtained.

Key Words: Mathematics education, problem posing, affective factors, systematic review.

REFERENCES

[1] K. E. Chang, L. J. Wu, S. E. Weng and Y. T. Sung, Embedding game-based problem-solving phase into problem-posing system for mathematics learning. Computers & Education, 58(2), 775-786, 2012.

[2] M. K. Ningsih, I. Sujadi and S. Subanti, Creative Thinking Processes of Vocational Students With High Learning Motivation In Mathematic Problem Posing. In University of Muhammadiyah Malang's 1st International Conference of Mathematics Education (INCOMED 2017) (pp. 192-196). Atlantis Press, 2018.

[3] M. Guo, F. K. S. Leung ve X. Hu, Affective determinants of mathematical problem posing: The case of Chinese Miao students. Educational Studies in Mathematics, 105, 367-387, 2020.

The Content Analysis of Concept Mapping Studies in Education: The Status of Concept Mapping Studies in Our Country in the Last 10 Years

Gülşah GEREZ CANTİMER [1] **and Sare ŞENGÜL**[2]
1 Ministry of Education, Sakarya/Serdivan IHO, gulsahgerez@subu.edu.tr
2 Marmara University, zsengul@marmara.edu.tr

ABSTRACT

The concept maps are graphical tools that show and organize relationships between concepts [1]. It is a set of semantic relations that enable information to be concretized and transformed into visuals [2]. It is used for many purposes in terms of revealing many skills such as critical thinking and reflective thinking, emphasizing the relationships between concepts, evaluating the deficiencies and facilitating the production of knowledge [3]. It emerges as rich learning opportunities in learning environments [4].

In this direction, a detailed examination of how concept mapping is used in educational fields, the general characteristics of the studies related to this field, their aims and results will create a holistic evaluation for the studies to be made in this field. In this context, the aim of the research is to analyze the current situation in our country in the last 10 years by analyzing the content of concept mapping studies in education. The theses on concept maps in the field of education completed between 2013-2022 were determined and examined. In this context, first of all, 312 theses were reached, the year limit and the subject of the research were clarified with keywords and 64 theses were examined. Studies that do not focus on concept maps in the summaries of the studies and that they are used only superficially with many methods and techniques are excluded from the scope.

According to the research findings; most of the studies consist of the master's theses (f=51). The number of the doctoral theses (f=13) was found to be less than the master's theses examined. Although the published studies vary according to the years, the most studies on concept mapping were reached in 2019 (f=20). The education fields in which the studies were conducted were respectively "science (f=12), mathematics (f=11), Turkish (f=9), biology (f=5), chemistry (f=4), computer and instructional technologies (f=3".), geography (f=3), religious education (f=3),

preschool (f=3), social studies (f=3), English (f=2), special education (f=2), Arabic teaching (f=1), physical education (f=1), education programs (f=1), Turkish literature (f=1)". When the concept maps were examined in particular for the mathematics lesson, it was determined that the subjects prepared/applied were mostly "polygons, fractions, rational numbers, exponential numbers". When the general characteristics of the studies are examined; it was determined that the sample group consisted mostly of secondary school students (f=28), and the quasi-experimental design with pre-test-post-test control group (f=31) was mostly preferred as a quantitative research method. The distribution of the studies according to the aims and results is examined in detail and suggestions will be made according to the outputs obtained.

Key Words: Education, concept mapping, content analysis.

REFERENCES

[1] J. D. Novak and A. J. Cañas, Theoretical origins of concept maps, how to construct them, and uses in education. *Reflecting education*, *3*(1), 29-42, 2007.

[2] R. Zorpuzan, Türkçenin yabancı dil olarak öğretiminde kavram haritalarının kelime öğretimi üzerine etkisi (Yüksek lisans tezi). Bolu Abant İzzet Baysal Üniversitesi, Eğitim Bilimleri Enstitüsü, Yabancı Dil Olarak Türkçe Öğretimi Bilim Dalı, 2019.

[3] J. L. Brinkerhoff and G. M. Booth, The effect of concept mapping on students' achievement in an introductory nonmajors biology. European International Journal of Science and Technology, 2(8), 43-46, 2013.

[4] J. Vanides, Y. Yin, M. Tomita and M. A. Ruiz-Primo, Concept maps. *Science Scope*, *28*(8), 27-31, 2005.

Problem Solving Processes of Preservice Mathematics Teachers with Geometer's Sketchpad: The Case ofSimson Line Theorem

Süleyman Güneş[1] and Rezan Yilmaz[2]
1 OndokuzMayis University, Graduate School of Educational Sciences,slymncc@gmail.com
2 OndokuzMayis University, Faculty of Education, rezzany@omu.edu.tr

ABSTRACT

In general, this study aims to examine the problem solving processes of preservice mathematics teachers in a Dynamic Mathematics Software (DMS) based environment. In particular, it aims to examine the process of solving Simson Line Theorem (or Simson-Wallace Theorem) as a problem in a DMS supported environment by considering a geometry problem case. So, the study was prepared in accordance with the qualitative research method and was designed as case study (Yin, 2003), and the case was considered as Simson Line Theorem. The participants were preservice teachers in a department mathematics education at a faculty of education of a big university in Black Sea Region. Criterion sampling method, one of the purposeful sampling methods, was used to determine the participants. The main criterion was the preservice teachers' ability to use Geometer's Sketchpad (GSP) (Jackiew, 1991) from DMS. The reason for choosing this program was that the whole process was constructed by the preservice teachers (Laborde, Kynigos, Hollebrands and Strässer, 2006). In accordance with the problem of the research, an activity requiring problem solving by using Simson Line Theorem (Isaacs, 2000) was prepared in the GSP environment. The activity was carried out with three participants who were in 2nd grade, 3rd grade and 4th grade. The problem solving processes of the participants were recorded by audio/video recording and obtained as data. Descriptive analysis was used to analyze the data (Büyüköztürk, Kılıç-Çakmak, Akgün, Karadeniz and Demirel, 2019; Yıldırım and Şimşek, 2006). The problem solving processes of the participants in the GSP environment were examined according to the problem solving stages of Polya (1957).

The results of the study demonstrate that the participants who did not receive training on problem solving stages also had problem solving skills, and all participants applied the problem solving stages correctly. Especially in the stage of

devising a plan, the participants used the dragging feature of the GSP effectively. In the looking back stage as evaluation, it was determined that the participants who could not reach a solution realized that the problem was not understood by them correctly at the stage of understanding the problem and so they started the problem solving process again, and then they reached the solution correctly. All participants checked the correctness of the solution easily during the looking back stage as evaluation, thanks to the GSP. Unlike the proofs of the theorem in the literature, the participants also put forward different solutions due to the characteristics of the GSP.

Key Words: problem solving, preservice mathematics teachers, Geometer's Sketchpad, dynamic mathematics software, Simson Line Theorem.

REFERENCES

[1] Büyüköztürk, Ş., Kılıç-Çakmak, E., Akgün, Ö. E., Karadeniz, Ş. &Demirel, F. (2019). Scientific Research Methods. In Qualitative Research (pp.259-263). Ankara: Pegem Academy Publications.

[2] Isaacs, I. M. (2001). *Geometry for college students*. Providence, RI: American Mathematical Society.

[3] Jackiew, N. (1991). The Geometer's Sketchpad[Computer Program]. Berkeley, CA, USA:Key Curriculum Press.

[4] Laborde, C., Kynigos, C., Hollebrands, K., &Strässer, R. (2006). Teaching and learning geometry with technology. Handbook of research on the psychology of mathematics education, 275-304.

[5] Polya, G. (1957). How to solve it? (2nd ed.). New York: Anchor Books.

[6] Yıldırım, A. &Şimşek, H. (2006). Qualitative Research Methods in the Social Sciences (8th ed.). Ankara: Seçkin Publications.

[7] Yin, R. K. (2003). Case study researches, design and methods (3rd ed.). California: Sage Publications.

Examining the Processes of 7th Grade Students' Understanding of the Concept of Arithmetic Mean in the Framework of 3E Learning Model: Within the Scope of Open-Ended Task

Sare Şengül [1], Mehmet Erçoban [2]
1 Department of Mathematics and Science Education, Atatürk Faculty of Education, Marmara University, 34722, Istanbul, Turkey, zsengul@marmara.edu.tr
2 Department of Mathematics and Science Education, Atatürk Faculty of Education, Marmara University, 34722, Istanbul, Turkey, ercobanmehmet@gmail.com

ABSTRACT

It is known that students have difficulty in structuring mathematical concepts at every stage of school life due to their abstract nature. Although the concept of arithmetic mean is widely used in daily life, it is one of the concepts that students have difficulty in conceptualizing and frequently make procedural mistakes in problem solving. The fact that the students did not encounter tasks in which they would structure the concept of arithmetic mean is among the reasons for their difficulties. For this reason, it is crucial for teachers to bring mathematical tasks that will help students structure their own conceptual knowledge into the classroom environment. In this context, it may be useful for teachers to make use of open-ended tasks, which is one of the types of mathematical tasks [1].

In line with the stated reasons, the purpose of the study is to examine the 7th grade students' understanding of the concept of arithmetic mean in the framework of the 3E learning model within the scope of an open-ended task. The study was conducted with sixteen 7th grade students in Istanbul. The activity was carried out as a small group work of 5-6 students with open-ended task named "Looking for Three More People". While designing the task, the "3E learning model" consisting of "exploration", "term introduction/explanation" and "concept application/expansion" phases was utilized [2]. Before starting the practice of the task, the goals of the lesson were determined by the researchers, and then the practice was started by taking expert opinions. During the practice of the open-ended task by the teacher, the "5-Phase Practices Framework" was used in order to progress and observe the students' responses and mathematical understanding as a whole [3].

In the "anticipating" phase, which is the first phase of the "5-Phase Practices Framework", the teacher determined student responses, solution methods and

difficulties that might be experienced in the task. In addition, students were carefully monitored during the activity and necessary in-class notes were taken. In some parts of the second phase of the practices framework, the "monitoring" phase, students were contacted and discussions were held to support their explorations. In the third and fourth phases of the framework, "selecting" and "sequencing", students and presentation order were determined. In the last phase, "connecting", the teacher helped the students make connections between their solutions and the mathematical goals of the lesson and introduced the terms. The application phase of the arithmetic mean concept was started for the students to practice and the study was concluded. The data obtained in the study were evaluated by descriptive analysis. The findings of the study show that the application of the 3E learning model with an open-ended task was effective in students' structuring of the concept of arithmetic mean.

Key Words: Arithmetic mean, open-ended task, 3E learning model, 5-phase practices framework.

REFERENCES

[1] P. Sullivan, D. Clarke and B. Clarke, Teaching with Tasks for Effective Mathematics Learning, New York, Springer, 2013.

[2] A. E. Lawson, Science Teaching and the Development of Thinking, Belmont, CA: Wadsworth, 1995.

[3] M. Smith and M. K. Stein, Five Practices for Orchestrating Productive Mathematical Discourse, Reston, VA: National Council of Teachers of Mathematics, 2011.

Attitudes and Views of Secondary School Students towards Problem Solving

Hasan Yasin TOL [1], **Selin ÇENBERCİ** [2]
1 Necmettin Erbakan University, Institute of Educational Sciences, Konya, Turkey, hasanyasintol@gmail.com
2 Necmettin Erbakan University, Ahmet Kelesoglu Education Faculty, Department of Mathematics Education, Konya, Turkey, scenberci@erbakan.edu.tr

ABSTRACT

Problem solving is an indispensable part of mathematics teaching and mathematics programs. It can be stated that understanding mathematical knowledge in mathematics teaching is achieved through problem solving. For this reason, it is emphasized that problem solving should be included in the teaching process and used effectively (NCTM, 2000). Therefore, providing problem solving skills to students has become one of the most important goals of mathematics teaching (MoNE, 2018). Effective learning environments with discussions that improve conceptual understanding lead students to think, improve their problem solving skills. Teachers have an important role in the design of such teaching environments. In this process, teachers should give students the opportunity to express their ideas, explain a topic with its reasons, and choose their own strategies and methods. In addition, teachers should carry the problems that will enable them to think differently to the learning environment. Therefore, the problem solving process should not be applied as a procedure where students are told what to do (Van de Walle, Karp and Bay-Williams, 2021). The realization of all these and the good management of the process requires knowing the attitudes and views of the students about problem solving. Thus, students' attitudes and thoughts about problem solving will give the teacher an idea about how to design the learning process. In this context, in this study, it was aimed to determine the attitudes of secondary school students about problem solving and to get their views. The "Mathematics Problem Solving Attitude Scale" developed by Çanakçı (2008) and open ended questions prepared by the researchers were used to collect the data of the study. The sample of the research consists of fifth and seventh grade students studying at a public school in Konya. In the study, students' attitudes towards problem solving were compared according to grade level and gender variables. The data obtained from the open ended questions were analyzed with the descriptive analysis method.

Key Words: Problem solving, attitude, 5^{th} and 7^{th} grade students.

REFERENCES

[1] National Council of Teachers of Mathematics, (2000). Standarts for school mathematics: problem solving, national council of teachers of mathematics, Reston, VA.

[2] MoNE, (2018). Mathematics teaching program (1,2,3,4,5,6,7 and 8^{th} grades), Republic of Turkey Ministry of National Education, Ankara.

[3] Van de Walle, J. A., Karp, K. S. & Bay-Williams, J. M. (2021). Elementary and Middle School mathematics: Teaching Developmentally (Dr. Soner Durmuş Trans.). Nobel Akademik Publishing, Ankara.

[4] Çanakçı, O. (2008). The development and the evaluation of mathematics problem solving attitude scale. Doctoral thesis, Institute of Educational Sciences, Marmara University, İstanbul.

Predictors of Mathematics Learning Disability (Dyscalculia)

Serkan Coştu[1] and Lena Lindenskov[2]
1 Kafkas University Dede Korkut Education Faculty, Dr. Res Asst., serkancostu@gmail.com
2 Aarhus University, Danish School of Education, DPU, Assoc. Prof., lenali@edu.au.dk

ABSTRACT

According to the literature, there is a need to define the predictors of dyscalculia to be diagnosed properly. This paper is aiming to determine the framework of predictors to define the person who has dyscalculia (MLD). It will be carried out as a selective meta-analysis of the literature due to its nature. The databases selected by some research were Cinahl and Academic Research Complete. Five papers were selected for each level of the framework developed by Frith (2001) which had four levels. These papers will be presented and also discussed in the context of the framework based on the work of Frith (2001) which will help us to classify all the predictors related to these levels in the model. It can be concluded that the causes of MLD are varied. Its origins come from a set of complex disorders like genetic factors, developmental delays, experiential limitations, language problems, cognitive weaknesses (perceptual, motor, memory), anxiety and unfavorable attitudes, and also inappropriate instructional practices (Haskell, 2000). Because of that, there are some (Mazzocco and Myers, 2003; Mazzocco, 2005) complexities in identifying dyscalculia. It can be seen that the framework based on the work of Frith (2001) that is used to represent results is more or likely adequate to understand the predictors of dyscalculia. This type of framework can be used to define the diagnostic tests to guide the researchers studying dyscalculia. However, it is also a requirement to refine this model of Frith like the attempt of Lagae (2008). Furthermore, the author's attempt should be regarded as an attempt to fulfill this idea of developing the perfect model. Moreover, the literature should be extended to cover all the predictors, and these predictors should be discussed deeply to clarify their effects on dyscalculia. And later, it is also possible to represent and recommend some diagnostic tools to examine these predictors.

Key Words: Meta-Synthesis, Mathematics Learning Disability (MLD), Dyscalculia, Framework, Predictors.

REFERENCES

[1] Frith U., (2001). 'What Framework Should We Use for Understanding Developmental Disorders?', Developmental Neuropsychology, 20: 2, 555 — 563

[2] Haskell S.H., (2000). The determinants of arithmetic skills in young children: some observations. European Child & Adolescent Psychiatry 9: II/77-II/86

[3] Mazzocco M.M.M. and Myers G.F., (2003). Annals of Dyslexia, Vol.53, 2003.

[4] Mazzocco M.M.M., (2005). Challenges in Identifying Target Skills for Math Disability Screening and Intervention. J Learn Disabil 2005; 38; 318.

[5] Lagae L., (2008). Learning Disabilities: Definitions, Epidemiology, Diagnosis, and Intervention Strategies. Pediatr Clin N Am 55 (2008) 1259–1268.

The Effect of PASS Theory Activities Used in the Learning Environment on the Development of Number Sense: A Pilot Study

Büşra Yaşar [1], Müjdat Karadağ[2], Ceren Ercan[3] and Serkan Coştu[4]
1 Kafkas University Dede Korkut Education Faculty, Res Asst., bsrysr314@gmail.com
2 Ministry of National Education, Teacher, mujdatkaradag@hotmail.com
3 Ministry of National Education, Teacher, cerenercan35@gmail.com
4 Kafkas University Dede Korkut Education Faculty, Dr. Res Asst., serkancostu@gmail.com

ABSTRACT

The research is a pilot study conducted to examine the effects of activities developed according to PASS theory on the development of number sense in preschool students. In this context, within the scope of the research, it is investigated the level of number sense of the students in the pre-school period (3-6 years old) and whether the learning environment created with the help of the activities prepared based on PASS theory affects the student's number sense levels. For this reason, it was carried out with the "One-Group Pretest-Posttest" model, which is one of the experimental designs before the experiment. In this direction, the study was carried out in a kindergarten, which consists of two classes with an average socioeconomic level. Pretest and posttest data were collected from a total of 25 students. However, the research findings were presented based on the data of 8 students in total, who fully participated in all processes. With the help of the teacher of the class, an implementation developed according to the PASS Theory to improve the number sense by using activities called "Sticks" and "Cards". In this direction, the "Assessing Number Sense Test" was applied individually by expert educators before and after the implementation. The study, which was carried out for two classes, was completed in a total of 6 weeks. The obtained data were analyzed and presented descriptively using test scores, comparing the mean scores and standard deviations.

As a result of the study, it was revealed that the pre-test results vary depending on the age in terms of the number sense skills of the preschool students. In addition, it can be said that the number sense scores obtained by preschool students are more positive in the post-test, therefore, the learning environment in which activities developed based on PASS theory is effective in developing number sense.

Key Words: Early Childhood, Preschool Students, Mathematics Education, Number Sense, PASS Theory, Activity Development

REFERENCES

[1] İvrendi, A. (2011). Influence of self-regulation on the development of childrens number sense. Early Childhood Educ Journal. 39, 239-247.

[2] İvrendi, A. (2016). Choice-driven peer play, self-regulation and number sense. European Early Childhood Education Research Journal, 24(6), 895-906.

[3] Lago, Rachel M. & Diperna, James C. (2010). Number sense in kindergarten: A factor-analytic study of the construct. School Psychology Review, 39(2), 164-180.

[4] Şengül, Sare ve Gülbağcı Dede, Hande (2013). Sayı hissi bileşenlerine ait sınıflandırmaların incelenmesi. The Journal of Academic Social Science Studies, 6(8), 645-664.

[5] Ün, D. (2020). PASS Teorisi Üzerine Genel Bir Literatür Taraması. İZÜ Eğitim Dergisi, 2(3), 84-100.

Investigation of Debugging Skills in Computational Thinking in the Technology-Enriched Mathematical Modelling Cycle

Süleyman Emre AKTAŞ[1], Çağlar Naci HIDIROĞLU[2]
1 Istanbul, Turkey, Ministry of National Education (Turkey), emre.aktas961@gmail.com
2 Denizli, Turkey, Pamukkale University, chidiroglu@pau.edu.tr

ABSTRACT

In the 21st century, it is considered important to develop students' skills such as problem solving, critical thinking, estimation, verification, logical and spatial thinking, and coordination [1, 2, 3, 4]. It is stated that mathematical modelling problems are effective in the development of such skills and it is possible to include them in teaching with mathematical modelling problems [5].

Mathematical modelling is a multifaceted problem-solving process that includes non-routine real-life problems, identifying knowns and unknowns, mathematizing real-life situations, constructing and relating multiple models related to real-life situations, interpreting the results obtained from the mathematical model in real-life situations, and verifying the solution [6, 7, 8, 9].

Wing [10] states that computational thinking, which was first introduced by Papert [11], is a basic skill that every individual should have, such as reading, writing, and arithmetic skills in the 21st century. Also, Ang [12] agrees with Wing's [13] view that "thinking like a computer scientist is more than knowing with programming", that he tells it is important to learn Scratch and Python, but they are not the ultimate goal, the more important thing is to look at how computers and information processing structures are used to solve problems.

In this research, it is aimed to examine the mental actions related to the debugging skill in the theoretical framework of computational thinking that Maharani et al [14], in the technology-aided mathematical modelling process created by Hıdıroğlu [9]. In this context, two mathematical modelling problems solved by three mathematics teaching students were analyzed according to the theoretical frameworks of mathematical modelling and computational thinking.

In this study, only debugging skills from the computational thinking skills (algorithmic, decomposition, generalization, abstraction, debugging) found in

theoretical framework of Maharani et al. [14] were taken into consideration. Debugging skill is expressed as identifying, disposing and correcting errors in the solution process [14].

In this study, debugging skill emerged in the basic steps of interpretation, verification and revision in the technology-aided mathematical modelling process. The ability to debug in the basic step of interpretation emerged in mental actions to determine the correctness and falsity of the real-life solution and results found in terms of the problem situation. In this step, the solvers judged whether the results they found were logical with the given data in the problem situation. Ability to debug the validation basic step; This was revealed when solvers-controlled operations, thoughts, and steps and compared them with their experience-based estimates or measurements of real-life outcomes. The ability to debug in the basic step of revision was revealed when the solver detected and corrected the wrong operations and developed different solution strategies as a result of the error found.

Key Words: Mathematical modelling, computational thinking, debugging.

REFERENCES

[1] Ananiadou, K., & Claro, M. (2009). *21st Century Skills and Competences for New Millennium Learners in OECD Countries*. https://www.oecd-ilibrary.org/content/paper/218525261154

[2] Milli Eğitim Bakanlığı [MEB]. (2018). *Matematik dersi öğretim programı (İlkokul ve Ortaokul 1,2, 3, 4, 5, 6, 7 ve 8. Sınıflar)*.

[3] National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics*.

[4] Partnership for 21st Century Skills [P21]. (2010). *Framework for 21st century skills*.

[5] Asempapa, R. S. (2015). Mathematical modeling: Essential for elementary and middle school students. *Journal of Mathematics Education*, *8*(1), 16–29.

[6] Blum, W., & Borromeo Ferri, R. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application*, *1*(1), 45–58.

[7] Blum, W., & Niss, M. (1989). Mathematical problem solving, modeling, applications, and links to other subjects—State, trends and issues in mathematics instruction. In M. Niss, W. Blum, & I. Huntley (Eds.), *Modeling applications and applied problem solvings* (pp. 1–19). Halsted Press.

[8] Galbraith, P., & Stillman, G. (2006). A framework for identifying student blockages during transitions in the modelling process. *Zentralblatt Für Didaktik Der Mathematik*, *38*(2), 143–162.

[9] Hıdıroğlu, Ç. N. (2015). *Teknoloji destekli ortamda matematiksel modelleme problemlerinin çözüm süreçlerinin analizi: Bilişsel ve üstbilişsel yapılar üzerine bir açıklama* [Doktora Tezi]. Dokuz Eylül Üniversitesi Eğitim Bilimleri Enstitüsü.

[10] Wing, J. (2006). Computational thinking. *Communications of the ACM*, *49*(3), 33–35.

[11] Papert, S. (1996). An exploration in the space of mathematics educations. *International Journal of Computers for Mathematical Learning*, *1*(1), 95–123.

[12] Ang, K. C. (2020). *Computational thinking as habits of mind for mathematical modelling*. https://repository.nie.edu.sg/bitstream/10497/22962/1/ATCM-2020-126.pdf

[13] Wing, J. (2008). Computational thinking and thinking about computing. *Philosophical Transactins of The Royal Society of London A Mathematical, Physical and Engineering Sciences*, *366*(1881), 3717–3725.

[14] Maharani, S., Kholid, M. N., Pradana, L. N., & Nusantara, T. (2019). Problem solving in the context of computational thinking. *Journal of Mathematics Education*, *8*(2), 109–116. https://doi.org/10.22460/infinity.v8i2.p109-116.

Discursively Examination of 8th Grade Students' Geometric Thinking Levels

Ferdag CULHAN[1], Emine Gaye CONTAY[2]

1 Alleben Mah. Mareşal Fevzi Çakmak Bul. No:10 Kat:3 Daire:15 / Şahinbey / Gaziantep, ferdaculhan0724@gmail.com
2 Pamukkale Üniversitesi Eğitim Fakültesi Matematik ve Fen Bilimleri Eğitimi Bölümü Matematik Eğitimi ABD Kınıklı Kampüsü Denizli, gayeermec@gmail.com

ABSTRACT

The aim of this study is determine the geometric thinking levels of 8th-grade students and to examine the geometric thinking levels of these students in-depth by revealing the differences discoursively. The research was carried out with 8th-grade students in a public school in Gaziantep. Geometric thinking levels of a group of 8th-grade students consisting of 54 students were determined, and then interviews were conducted with six students who were at the first, second ant the third geometric thinking levels. The study was defined as case study.

In the study, the data were collected in two sections. In the first section, the "Global Van Hiele Questionnaire," which was translated into Turkish was applied to determine the geometric thinking levels of 8th-grade students. In the second section, during the interviews with six selected students, they were asked to complete "Global Van Hiele Questionnaire".

In order to reveal the mathematical discourses of the 8th-grade students, the responses of the students to the "Discourse Analysis Interview Form" were analyzed by the content analysis method. The mathematical discourses of 8th-grade students were classified within the four properties (word use, visual mediators, routines, endorsed narratives) according to Sfard's (2008) "Commognitive" theory.

The results of the study showed that the geometric thinking levels of the 8th-grade students were lower than expected (third geometric thinking level). It has been determined that the most common responses were at the level of first and below. It was determined that the geometric thinking levels of the students in the sub-learning area of geometric objects were higher than the other sub-learning areas of geometric thinking levels. It was determined that one of the students at the first level of geometric thinking was able to explain the properties of geometric shapes in a more

detailed way. It has been realised that the learners who have been at the second geometric thinking level could generally explain the aspects of the geometric shapes. Also, the learners at the second geometric thinking level could use expressions at the third geometric thinking level. However, although they could use expressions at the third geometric thinking level, it has been determined that there were information deficiencies and mistakes about the properties of geometric shapes. On the other hand, it was determined that the students at the third level of geometric thinking level generally used expressions suitable for their levels, but they lacked knowledge and made mistakes about the properties of some shapes. Besides, it was concluded that the mathematical discourses of the students at the same geometric thinking levels differed regarding geometric shapes they explained. It was also found that even if the students were at same geometric thinking levels, they did not make the same level of progress about certain geometric shapes.

Key Words: Van Hiele, Geometric Thinking Levels, Commognitive Framework, 8TH Grade, Mathematical Discourses

REFERENCES

[1] Patkin, D. (2014). Global van Hiele (GVH) Questionnaire as a tool for mapping knowledge and understanding of plane and solid geometry. *Journal of the Korean Society of Mathematical Education. Series D: Res. Mathe. Educ*, *18*(2), 103-128.

[2] Sfard, A. (2008). Thinking as Communicating: human development, the growth of discourses, and mathematizing. Cambridge: Cambridge University

Model Eliciting Activity Experiences of Primary School Teachers: A TÜBİTAK Project(TM2İ)

Dr. İsmail KAYGISIZ[1], Dr. Öğrt. Üyesi İbrahim ÇETİN[2], Mustafa ALTUNTAŞ[3], Şeyma KARAN[4], Halil İbrahim CANLI[5]

1 Selçuklu İlçe MEM, MEB, ismailkaygisiz1981@gmail.com
2 Necmettin Erbakan Üniversitesi Ahmet Keleşoğlu Eğitim Fakültesi Matematik ve Fen Bilimleri Eğitimi Bölümü, ibrahincetin@konya.edu.tr
3 Karatay Bilim ve Sanat Merkezi MEB, altuntas2012@gmail.com
4 Ali Fuat Cebesoy İlkokulu Ataşehir İstanbul MEB, 7renk.yedi@gmail.com
5 Sille Tabiat Okulu Selçuklu konya MEB, halilcanli_1542@hotmail.com

ABSTRACT

Mathematical modeling is defined as a process that includes the analysis of a real-life problem with mathematical methods by transferring it to the world of mathematics (Borromeo-Ferri, 2010; Bukova-Guzel, 2016; Maaß, 2006). According to Maaß (2006), in modeling activities, students need to make sense of the real-life situation and express it in mathematical language, analyze and interpret the information given about the situation, select the necessary data and associate the solution with the real -life situation within the framework of these data. He also states that modeling activities are more effective than traditional word problems for the discovery of mathematics in real life and the mathematical development of students (Maaß, 2006). The necessity of associating daily life situations with mathematics has been frequently emphasized by mathematics educators in recent years. It is important to relate mathematics to daily life situations with mathematical modeling activities that are applied to students in the primary school period. The biggest difficulty encountered in mathematics education when students work with daily life problems is that students cannot transfer the knowledge they learned during the problem-solving phase to daily life (Altun, 2018). In this context, creating activities that teachers will implement in their classrooms will ensure that the mathematics lesson is taught effectively and permanently.

In this study, teachers' experiences of creating mathematical modeling within the scope of a TÜBİTAK project were collected with a semi-structured interview form. The obtained data was analyzed by content analysis. According to the data obtained,

the teachers stated that they had difficulties in the modeling activity they first tried to create, but it was more understandable to create modeling over time.

Key Words: Mathematical modeling, primary school, TÜBİTAK project.

REFERENCES

[1] Altun, M. (2018). Eğitim fakülteleri ve ilköğretim öğretmenleri için matematik eğitimi (21. Baskı). Bursa: Aktüel Yayınevi.

[2] Borromeo Ferri, R. (2010). On the influence of mathematical thinking styles on learners' modeling behaviour. Journal für Mathematik didaktik. 31 (1), 99-118.

[3] Bukova-Güzel, E. (2016). Matematik eğitiminde matematiksel modelleme. (1.Baskı). Ankara: Pegem Akademi.

[4] Maaß, K. (2006). What are modelling competencies? The International Journal on Mathematics Education, 38 (2), 113-142.

The Effect of Singapore Model and Worked-Out Examples Methods on Students' Success Level on Solvingg Word Problems Related to Numbers

Esengül YILDIZ [1], **Prof. Dr. Seniye Renan SEZER** [2]
Ankara Ölçme Değerlendirme Merkezi, esengulerensoy@gmail.com
Ankara Üniversitesi Eğitim Bilimleri Fakültesi, rsezer@ankara.edu.tr

ABSTRACT

In this study, the effect of the worked-out examples method and Singapore model method on sixth-grade students' problem solving skills were separately analyzed. To this end, pre- and post-test results along with a control group were utilized through a semi-experimental design. Worked-out examples method is a teaching method based on cognitive load theory. This method helps to reduce cognitive load. In the worked-out examples method, the correct solution steps for previously determined problems are presented. Singapore Model Method is a method that helps students to visualize the connections between the variables in the problem. This is done via representation of these variables through rectangles of different sizes. This method has been utilized in the Singaporean curriculum for primary school students in the solving word problems. The Singapore Model Method is based on Mayer's two-stage problem-solving model. This model combines schema theory and problem solving theory. The researcher developed a pre and post-problem solving achievement test on word problems based on numbers at the sixth-grade level. In developing this test many-facets Rasch measurement model based on Item Response Theory was used. The subjects of the study were 61 sixth-grade students. The study involved three sixth-grade classrooms at a public school. After the pre-test was conducted, two classrooms were randomly chosen as experimental groups and one as a control group. After the experimental and control groups were determined, one of the two problem solving methods were utilized for instruction in the experimental groups; whereas, the control group was instructed in the usual fashion. Following the intervention, in all three groups, the post-test, which was identical to the pre-test, was conducted. Significant differences in the pre-test and post-test scores were analyzed with one-way covariance analysis (ANCOVA). In the analysis it was indicated that when pre-test results were considered, experimental group 2, using the worked-out examples method, had a statistically relevant advantage in their post-test scores.

Key Words: Singapore model method, worked-out examples method, problem solving.

REFERENCES

[1] A.A. Darabi, D.W. Nelson & F. Paas, Learner Involvement in Instruction on a Complex Cognitive Task: Application of a Composite Measure of Performance and Mental Effort, Journal of Research on Technology in Education (2007),40(1), 39-48.

[2] J. J. G. Van Merriënboer, R. E.Clark and M. B. M. de Croock, Blueprints for complex learning: The 4C/ID-Model, Educational Technology Research and Development (2002),50(2):39-61.

[3] K. Mahoney, Effects Of Sıngapore's Model Method On Elementary Student Problem Solvıng Performance: Sıngle Subject Research (Published doctoral dissertation), College of Professional Studies Northeastern University, Boston, Massachusetts, 2012.

[4] R. E. Mayer, Thinking, problem solving, cognition, New York: W.H. Freeman and Company, 1983.

[5] S. F. Ng and K. Lee, The model method: Singapore children's tool for representing and solving algebraic word problems, Journal for Research in Mathematics Education (2009),40(3), 282–313.

The Effects of Mathematics on Human Intelligence-I

İbrahim GÜNALTILI[1]

1. Eskişehir Osmangazi University Faculty of Science igunalti@ogu.edu.tr

ABSTRACT

In this article, the differences between primary, secondary, high school and university students' understanding and learning of a mathematical concept will be determined and some ways to achieve 100% success on this concept will be given. The most important of these ways, "Full Understanding-Full Learning Chain" (TAÖ.Z.) will be explained. In addition, in this study, the mathematical analysis of human common thinking, mental power and sublimity is presented in detail..

Key Words: Mathematices,Education.

REFERENCES

[1] Ibrahim Gunaltili, Application of graph theory to writer recognition of Turkish characters, preprint
[2] RE Moore, Interval analysis (Englewood Cliffs, NJ: Prentice-Hall, 1966).
[3] P.O. Bishop, Neurophysiology of binocular vision, in J. Houseman (Ed.), Handbook of physiology, 4 (New York: Springer-Verlag, 1970) 342-366.
[4] Ibrahim Gunaltili, On Strategies for Effective Mathematics Education, preprint
[5] WJ Book, Modeling design and control of flexible manipulator arms: A tutorial review, Proc. 29th IEEE Conf. on Decision and Control, San Francisco, CA, 1990, 500-506.

Some Factors Affecting Mathematics Success with Problem Solving in Computer Based Assessment: TIMSS 2019

Burcu Parlak [1]
Milli Eğitim Bakanlığı, üye değil, enbaparlak@gmail.com

ABSTRACT

Although many studies on 21st century skills have attracted attention in recent years, there are different classification studies on what 21st century skills are in the literature. One of the classifications accepted in the literature is the P21 classification. Here, 21st century skills are the three main skills referred to i) learning and innovation skills, ii) knowledge, media and technology skills, and iii) life and career skills. In this classification, learning and innovation skills (4C- critical thinking, communication, collaboration, creativity); It consists of four skills: critical thinking and problem solving, communication, cooperation and creativity (Kylonen, 2012; Partnership for 21st Century Learning, 2007). Problem solving skill is defined as the mental processes necessary for the realization of a goal or overcoming a problem (Haladyna, 1997). In a different classification study of 21st century skills, it is seen that problem solving skills are included in cognitive skills (eg, critical thinking, problem solving, creativity) (2010; Kylonen, 2012).

Another issue related to 21st century skills is the measurement of these skills. It is seen that rating scales, performance evaluation and simulation, measurement tools containing different item types (such as multiple choice, computer-assisted and open-ended items) are used. The computer aided environment allows the use of applications such as computer simulations based on real situations from these approaches, which gives the opportunity to measure cognitive skills very close to reality.

Cognitive skills mean skills developed in school and generally measured with standardized tests, in large-scale national and international (PISA, TIMSS) assessments, in content areas such as mathematics and science (Kylonen, 2012).

TIMSS 2019, one of these applications, has started to evaluate problem solving skills in the computer environment.

It is seen that the measurements and evaluations made in the computer environment have just started in Turkey and are limited in number. Therefore, the findings of this study are considered to be important.

In this study, analyzes were made by using the achievement scores obtained from the eighth mathematics tests performed in TIMSS 2019 computer environment and applied for problem solving skills, and data on the students' questionnaire items. In the analyzes made, it was determined that the students' own computers or tablets and internet connection made a statistically significant difference in the findings of the problem-solving mathematics questions applied in the computer environment. It has been determined that the mathematics success average of the students who say they have a computer or tablet is 507,91, and the students who do not have a 443,11. Similarly, it is seen that the mathematics success average of the students who state that they have an internet connection is 504,65, while the students who do not have an internet connection are 456,25. In both findings, it is seen that the success of the students who have the relevant education and training tool in mathematics questions based on problem solving applied in the computer environment is quite high, and the difference between the average score of the students who do not have it is statistically significant.

Key Words: Mathematics achievement, problem solving, computer-based applications, TIMSS.

REFERENCES

[1] Haladyna, T. M. (1997). Writing test item to evaluate higher order thinking. USA: Allyn and Bacon

[2] DiCerbo, K. (2014). Assessment and teaching of 21st century skills. *Assessment in Education: Principles, Policy and Practice, 21*(4), 502-505. doi:10.1080/0969594X.2014.931836;

[3] Kylonen, P. C. (2012). *Measurement of 21st century skills within the common core state standards*. Paper presented at the Invitational Research Symposium on Technology Enhanced Assessments, May 7-8.

[4] Lai, E. R. and Viering, M. (2012). *Assessing 21st century skills: Integrating research findings*. Vancouver, B.C.: National Council on Measurement in Education.

[5] Partnership for 21st Century Learning (P21). (2007). *Framework for 21st century learning*. http://www.p21.org/our-work/p21.

Gamification in Computer Aided Mathematics Education: Classcraft

Berna Aygün [1]
1 Süleyman Demirel University, Mathematics Education, bernaaygun@sdu.edu.tr

ABSTRACT

As in every field, education is constantly evolving and changing in line with today's needs. One of the changes in education is the use of games in the education system. It can bring together games and education under the name of “gamification”. The most accepted definition of gamification is the use of game design elements in non-game environments (Deterding et al., 2011). Gamification is used to encourage learning and to ensure the continuity of learning. With the help of technology, many educational games and digital education materials can be created. One of the gamification technological tools is Classcraft. Classcraft is an online platform that allows the design and implementation of the gamification environment. The purpose of Classcraft is to turn the classroom into a role-playing scenario, giving students a variety of tasks and responsibilities. In this gamification process, students earn points that enable them to gain power, level up and progress in the game. In this study, it is aimed to examine the opinions of elementary mathematics teacher candidates about the gamification process in the ClassDojo application used in computer aided mathematics education. This study was designed with the qualitative research method. Teachers candidates were involved in the gamification process for 14 weeks, and semi-structured interviews were conducted with the pre-service teachers at the end of the process. In line with the data obtained, the views of the pre-service teachers on the gamification process and its elements were analyzed. As a result of the findings of the study the interest and motivation of the teacher candidates towards the lesson increased in the the Gamified Computer Aided Mathematics Education via Classcraft.

Key Words: computer Aided Math Education, classcraft, gamification.

REFERENCES

[1] Deterding, Sebastian, et al. "Gamification. using game-design elements in non-gaming contexts." *CHI'11 extended abstracts on human factors in computing systems*. 2011. 2425-2428.

Figures Placed Between Geometric Patterns in Medieval Anatolian Turkish-Islamic Architecture

Fatmagül Saklavcı[1]
1 Doctor. Cumhuriyet University,
Fsaklavci@gmail.com

ABSTRACT

In our study, an evaluation of the figures placed among the geometric ornaments in the structures of the Middle Ages Anatolian Seljuk period has been made. In Anatolia, which has become a multicultural and multinational, a bridge between Asia and Europe since the 11th century, decoration and building elements with many different origins took place in the Anatolian Seljuk architecture (Kuban 1964: 45-46). The vegetal and geometric patterned strips, inscription and muqarnas strips and moldings, ornamental strips and siege arches, gülbezek and hovels designed especially for the portal frame, which are the most striking parts of the buildings, are the most important examples of Seljuk stonework (Ögel 2006: 475). Although the number of geometric ornament strips surrounding the three sides of the portals in the buildings differ, they have the same symbolic meanings. Ornamental areas consisting of geometric shapes are usually designed with a grift or plain composition and mostly star forms. In some buildings, various figures are placed inside the ornamental elements such as stars and rosettes, which are among these ornaments (Önkol 2016: 116). Among these structures, Niğde Hüdavend Hatun Mosque and Bünyan Ulu Mosque have figures given in profile and with their whole bodies, depending on the branch, between the spirals in the form of geometric curves on the portal borders. On the portals of Niğde Alaeddin Mosque, Aksaray Sultan Han, Eskişehir Alemşah Vault, Aksaray Sultan Han, Denizli Akhan Caravanserai and Burdur Susuz Han, the figures are placed inside the rosettes between the geometric forms. These figures have different symbolic meanings together with geometric designs. In Anatolia in the Middle Ages, especially in the Seljuk period, elements related to the sky such as the moon, sun, planet and star, sometimes figures such as fish, dragon, lion, deer and bird, and sometimes in different rosettes, passionflower, flower and star forms together with protective and power are symbolized as elements (Alp 2009: 43).The universe was tried to be depicted together with the wheel of fortune symbols on the badges symbolizing animals and the sun (Esin 1972: 314-327). The sun reflects the heat and light given by God (Öney 1992: 36). Cosmic designs can be considered as an expression of protecting structures from evil and respecting religious places (Karamağaralı 1993: 261). In Anatolian Seljuks, badges are the most used motifs as eagle, moon, sun and planets and symbol of light. The moon and the sun are deified symbols that are respected in Anatolia. The moon, which rules the waters and rains, is a symbol of universal fertility, death and rebirth. Circular geometric designs point to an expression of infinity and an order of the universe (Ögel 1994: 95). Star forms symbolize the sultan, the representative of God on earth. Circle form, sphere, hobnail and rosettes, which represent infinity in architectural decorations, are thought to express religious and cosmological meanings such as god, sky and universe as small "sky dome" symbols (Yazar and

Doğan 2013: 239). Cosmic designs can be considered as an expression of protecting structures from evil and respecting religious places. It is possible that these forms symbolize the zodiac signs and planets, but also represent the sultan (Durukan 1993: 151-152).

Key Words: Geometric ornaments, Medieval Anatolian Seljuk period, figure.

REFERENCES

[1] D. Kuban, Anadolu Türk Mimarisinin Kaynak ve Sorunları, İstanbul, 1964.

[2] S. Ögel, Taçkapılar, Anadolu Selçukluları ve Beylikler Dönemi Uygarlığı, Kültür Bakanlığı Yayınları, Ankara, 2006.

[3] Ç. Ö. Tunç, Anadolu Selçuklu Dönemi Taçkapıları Süsleme Şeritlerinde Tezyinat, Pamukkale Üniversitesi İlahiyat Fakültesi Dergisi, 3/ 5, (2016), 114-131.

[4] Ö. Alp, Orta Asya'dan Anadolu'ya Kültürel Sembollere Giriş, Eflatun Yayınevi, Ankara, 2009.

[5] S. Ögel, Anadolu'nun Selçuklu Çehresi, Akbank Yay., İstanbul, 1994.

[6] T. Yazar and Doğan N. Şaman Doğan, Anadolu Selçuklu ve Beylikler Dönemi Mimari Süslemesinde Küre, Küre ve Koni Kesiti/Kabara, Rozet, Hacettepe Üniversitesi Türkiyat Araştırmaları Dergisi, (2013), 19: 221-245.

[7] E. Esin, Kün-ay, 7. Türk Tarih Kongresi, TTK Basımevi, Ankara, 1972.

[8] B. Karamağaralı, İçiçe Daire Motiflerinin Mahiyeti Hakkında, Sanat Tarihinde İkonografik Araştırmalar Güner İnal'a Armağan, Ankara, (1993), 249-270.

[9] A. Durukan, Ak Han'ın Süsleme Programı, Sanat Tarihinde İkonografik Araştırmalar Güner İnal'a Armağan, Ankara, (1993), 143-161.

[10] G. Öney, Anadolu Selçuklu Mimari Süslemesi ve El Sanatları. Türkiye İş Bankası Kültür Yay. Ankara, 1992.

Effects of Mathematics Lesson Activities Connected With Different Disciplines on Primary School Students

Şevket İlgün [1], **Bülent Nuri Özcan** [2]
1 Ministary of Education, lgnsevket@gmail.com
2 Faculty of Education, Manisa Celal Bayar University, bnozcan@gmail.com

ABSTRACT

Mathematical connection is one of the mathematical skills that should be taught to students (National Council of Teachers of Mathematics [NCTM], 2000). The ability to make connections emerges as the ability to associate mathematical expressions within themselves, with different fields or with daily life, and to establish connections between them (Bingölbali & Coşkun, 2016). In order for students to understand the concepts existing in mathematics, it is important to establish relationships between these mathematical concepts and to enable students to use these concepts in their lives and in other disciplines (MEB, 2009). The richer and stronger the network of relationships for mathematical concepts, the easier and more permanent meaningful learning occurs (Skemp, 1976). When the stated factors were evaluated, it was decided that the focus of this research would be to make connections with other disciplines in the mathematics course.

This research was carried out to reveal how mathematics lesson activities connected with different disciplines in primary school 3rd grade affect students' mathematics anxiety, mathematics lesson motivation and attitudes towards mathematics lesson. For this purpose, at the end of the research, it is aimed to reveal various application examples of activities connected with different disciplines that teachers can benefit from.

The research was carried out as an action research. Action research is a qualitative research design that is frequently applied to improve current situations or eliminate problems. Easily accessible case sampling method was used in the formation of the research group. 17 students in the 3rd grade of a primary school located in the Gölmarmara district center of Manisa province participated in the research.

Data collection tools in the research; “Mathematics Anxiety Scale for Primary School Children”, “Mathematics Lesson Motivation Scale for Primary School 3rd and 4th Grade Students”, “Attitude Scale Towards Adapted Mathematics Short Form” constitute semi-structured interview forms, student diaries and observation notes. Before the application, pre-interviews were made with the students and the scales were applied as a pre-test. After the application, interviews were made with the students and the scales were applied as a post-test.

The analysis of the quantitative data was done with the Wilcoxon test. The data obtained from the interview, diary and observation notes were analyzed with descriptive analysis.

As a result of the analysis, significant differences emerged between the scores of the students in the pretest and posttest of the anxiety, motivation and attitude scale. It has been revealed that the teaching process carried out with mathematics lesson activities connected with different disciplines reduces students' mathematics anxiety, increases students' motivation towards mathematics lessons and positively affects their attitudes towards mathematics.

Key Words: Teaching mathematics, mathematical connection, connections skill, connecting with different disciplines, primary school.

REFERENCES

[1] E. Bingölbali,and M. Coşkun, İlişkilendirme Becerisinin Matematik Öğretiminde Kullanımının Geliştirilmesi İçin Kavramsal Çerçeve Önerisi. Eğitim ve Bilim. (2016). 41(183): 233-249.

[2] Milli Eğitim Bakanlığı [MEB], İlköğretim Matematik Dersi 6-8. Sınıflar Öğretim Programı ve Kılavuzu. Devlet Kitapları Müdürlüğü Basımevi. Ankara, 2009.

[3] National Council of Teachers of Mathematics [NCTM], Principles and Standards for School Mathematics. Reston, VA: NCTM, 2000

[4] R. R. Skemp, Relational Understanding and Instrumental Understanding. Mathematics Teaching. (1976), 77: 20-26.

LISTS OF PARTICIPANTS OF ICMME-2022

Name	Institution Name	Country
Abdulhamit Kücükaslan	Ankara Yıldırım Beyazıt University	Turkey
Abdullah Akkurt	Kahramanmaraş Sütçü İmam University	Turkey
Adnan Karataş	Pamukkale University	Turkey
Ainur Temirkhanova	L.N. Gumilyov Eurasian National University	Kazakhstan
Akbota Abylayeva	L.N.Gumilyov Eurasian National university	Kazakhstan
Ali Aytekin	Pamukkale University	Turkey
Ali Filiz	Pamukkale University	Turkey
Ali Kurt	Pamukkale University	Turkey
Ali Gökhan	Kütahya Dumlupınar University	Turkey
Aliya Naaz Siddiqui	Maharishi Markandeshwar	India
Alp Arslan Kıraç	Pamukkale University	Turkey
Amiran Gogatishvili	Institute of Mathematics of the Czech Academy of Sciences	Czech Republic
Anoosha Qaisar	University of Verona Italy	Italy
Asuman Duatepe Paksu	Pamukkale University	Turkey
Aygül Bilgin	Haliç University	Turkey
Aynur Keskin Kaymakcı	Selcuk University	Turkey
Aysu İrem Sönmez	Hacettepe University	Turkey
Aysun Erdoğan Sert	Manisa Celal Bayar University	Turkey
Ayşe Nur Tunç	Çanakkale Onsekiz Mart University	Turkey
Ayşegül Daşcıoğlu	Pamukkale University	Turkey
Azhar Abek	L.N. Gumilyov Eurasian National University, Nur-Sultan	Kazakhstan
Aziz Ul İslam	University of Malakand Chakdara District Dir lower Khyber Pakhtunkhwa	Pakistan
Bakhytzhan Yegizbay	Astana İnternational University	Kazakhstan
Berna Aygün	Süleyman Demirel University	Turkey
Birgül Aksoy	Eskişehir Technical University	Turkey
Burcu Çalışkan Karakulak	Necmettin Erbakan University	Turkey
Burcu Parlak	Ministry of Education	Turkey
Büşra Kıral	İstanbul Aydın University	Turkey
Büşra Selen	Pamukkale University	Turkey
Canan Celep Yücel	Pamukkale University	Turkey

Canan Hazar Güleç	Pamukkale University	Turkey
Canay Aykol Kocakuşaklı	Ankara University	Turkey
Cansel Aycan	Pamukkale University	Turkey
Çağlar Naci Hıdıroğlu	Pamukkale University	Turkey
Deniz Elmacı	Dokuz Eylul University	Turkey
Deniz Güney	Eskişehir Osmangazi University	Turkey
Dilek Varol	Pamukkale University	Turkey
Ebru Mutlu	Pamukkale Üniversity	Turkey
Ebru Altıparmak	Erzurum Technical University	Turkey
Ebru Ateşok	Kırıkkale University	Turkey
Ebru Keskin	Kırıkkale University	Turkey
Ella Abylaeva	Kyrgyz-Turkish Manas University	Kyrgyzstan
Emel Karaca	Ankara Hacı Bayram Veli University	Turkey
Emel Çenet	Bozok University	Turkey
Emine Gaye Contay	Pamukkale University	Turkey
Erdem Kocakuşaklı	Sinop University	Turkey
Esengül Yıldız	Ankara University	Turkey
Esra Avcı Çongar	Cumhuriyet University	Turkey
Fatmagül Saklavcı	Cumhuriyet University	Turkey
Ferdağ Çulhan	Pamukkale University	Turkey
Firdevs Kazıcı	Pamukkale University	Turkey
Furkan Talha Tunç	Denizli Provincial Directorate of National Education	Turkey
Ghulam Rasool	Beijing University of Technology	China
Gül Karadeniz Gözeri	İstanbul University	Turkey
Gülay Koru Yücekaya	Selçuk University	Turkey
Güler Tuluk	Kastamonu University	Turkey
Gülnaz Boruzanlı Ekinci	Ege University	Turkey
Gülseli Burak	Pamukkale University	Turkey
Gülsen Pekince	Eskişehir Osmangazi University	Turkey
Gülşah Gerez Cantimer	Denizli Provincial Directorate of National Education	Turkey
Güner Topcu	Selcuk University	Turkey
Hamdullah Başaran	Süleyman Demirel University	Turkey
Hamide Soysal Demirci	Bozok University	Turkey

Handan Çerdik Yaslan	Pamukkale University	Turkey
Hasan Çakan	Pamukkale University	Turkey
Hasan Arslan	Erciyes University	Turkey
Hasan Hüseyin Sayan	Gazi University	Turkey
Hasan Yasin Tol	Necmettin Erbakan University	Turkey
Hatice Tozak	Denizli Provincial Directorate of National Education	Turkey
Hatice Nur Koç	Pamukkale University	Turkey
Havva Nur Öztürk	İstanbul University	Turkey
İbrahim Günaltılı	Eskişehir Osmangazi University	Turkey
İbrahim Çelik	Pamukkale University	Turkey
İlyas Haşimoğlu	Karabuk University	Turkey
İsmail Yaslan	Pamukkale University	Turkey
İsmail Kaygısız	Denizli Provincial Directorate of National Education	Turkey
İsmet Ayhan	Pamukkale University	Turkey
Jalolxon Nuritdinov	National University of Uzbekistan	Uzbekistan
Jan Zemlicka	Charles University	Czech Republic
Javanshir Hasanov	Azerbaijan State Oil and Industry University	Azerbaijan
Kemal Aydın	Selcuk University	Turkey
Kübra Özkan	Manisa Celal Bayar University	Turkey
Kübra Oymak	Pamukkale University	Turkey
Kürşat Canevi	Karamanoğlu Mehmetbey University	Turkey
M. Tamer Koşan	Gazi University	Turkey
Madi Muratbekov	Esil University	Kazakhstan
Malkhaz Bakuradze	Tbilisi State University	Georgia
Mehmet Erçoban	Marmara University	Turkey
Mehmet Ali Gökbaraz	Pamukkale University	Turkey
Melih Mert Oskay	Yildiz Technical University	Turkey
Merve Yakar	Pamukkale University	Turkey
Merve Görgülü	Karamanoğlu Mehmetbey University	Turkey
Miguel Ortega	Granada University	Turkey
Miraç Çetin	Baskent University	Turkey
Mithat Özçıkrıkcı	Selçuk University	Turkey
Moawwad El-Mikkawy	Mansoura University, Faculty of Science,Mansoura	Egypt
Muhammad Ramzan	Bahria University, Islamabad, Pakistan	Pakistan

Muhammed İbrahim Okcu	Pamukkale University	Turkey
Mukaddes Ökten Turacı	Pamukkale University	Turkey
Murat Beşenk	Pamukkale University	Turkey
Murat Altunbaş	Erzincan Binali Yıldırım University	Turkey
Mustafa Özkan	Gazi University	Turkey
Mustafa Aşcı	Pamukkale University	Turkey
Mustafa Yılmaz	Pamukkale University	Turkey
Mustafa Yıldız	Zonguldak Bülent Ecevit University	Turkey
Mustafa Gülfırat	Ankara University	Turkey
Mustafa Hicret Yaman	Mugla Sitki Kocman University	Turkey
Müjdat Ağcayazı	Aydin Adnan Menderes University	Turkey
Naglaa Nasser	Noor international school	Egypt
Necdet Güner	Pamukkale University	Turkey
Nefise Yüksel	Bozok University	Turkey
Nehir Keyik	19 Mayıs University	Turkey
Neslihan Kılıç Gürçay	Balıkesir University	Turkey
Nihal Yokuş	Karamanoğlu Mehmetbey University	Turkey
Nilay Akgönüllü Pirim	MATDER	Turkey
Nurettin Doğan	Selçuk University	Turkey
Nurzhan Bokayev	L.N. Gumilyov Eurasian National University, Nur-Sultan	Kazakhstan
Okan Bodur	Pamukkale University	Turkey
Orhan Özdemir	Tokat Gaziosmanpasa University	Turkey
Ömer Güleç	Pamukkale University	Turkey
Ömer Akgüller	Mugla Sitki Kocman University	Turkey
Özbuğ İlayda Öztürk	Anadolu University	Turkey
Özge Arıbaş	Zonguldak Bülent Ecevit University	Turkey
Özlem Girgin Atlıhan	Pamukkale University	Turkey
Pelin Yaprakdal Uzun	Burdur Mehmet Akif Ersoy University	Turkey
Péter Négyesi	Eszterházy Károly Catholic University	Hungary
Ramazan Duran	Afyon Kocatepe University	Turkey
Ramazan Sunar	Denizli Provincial Directorate of National Education	Turkey
Ramin Farzullazadeh	Lankaran State University	Azerbaijan
Rauf Amirov	Sivas Cumhuriyet University	Turkey
Reyhan Canatan İlbey	Başkent University	Turkey

Ruslan Surmanidze	Ivane Javakhishvili Tbilisi State University	Georgia
Ryskul Oinarov	L.N. Gumilyov Eurasian National University	Kazakhstan
Sare Şengül	Marmara University	Turkey
Sati Ceylan Oral	Burhaniye Science and Art Center	Turkey
Sawano Yoshihiro	Chuo University	Japan
Seçil Özekinci	Pamukkale University	Turkey
Semail Ulgen	Antalya Bilim University	Turkey
Sena Özen Yıldırım	Çanakkale Onsekiz Mart University	Turkey
Serel Madak	Pamukkale University	Turkey
Serkan Coştu	Kafkas University	Turkey
Serpil Salınan	Pamukkale University	Turkey
Serpil Halıcı	Pamukkale University	Turkey
Sevim Durak	Sivas Cumhuriyet University	Turkey
Shabnur Alizade	lankaran State University	Azerbaijan
Shilpa Kapoor	Centarl University	India
Siyat Nur Noyan	Selçuk University	Turkey
Süleyman Cengizci	Antalya Bilim University	Turkey
Süleyman Aydınyüz	Pamukkale University	Turkey
Süleyman Güneş	Ondokuz Mayıs University	Turkey
Süleyman Emre Aktaş	Pamukkale University	Turkey
Şahin Ceran	Pamukkale University	Turkey
Şaziye Ece Özdemir	Pamukkale University	Turkey
Şerife Yılmaz	Burdur Mehmet Akif Ersoy University	Turkey
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